USING GAME THEORY TO ANALYZE SUSTAINABILITY AND DECISION MAKING OF FIRMS

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Introduction
Sustainability refers to meeting our needs without compromising the ability of future generations to meet their needs. Environmental sustainability specifically refers to our needs for natural resources and to decrease the negative impact of human activities on the environment.

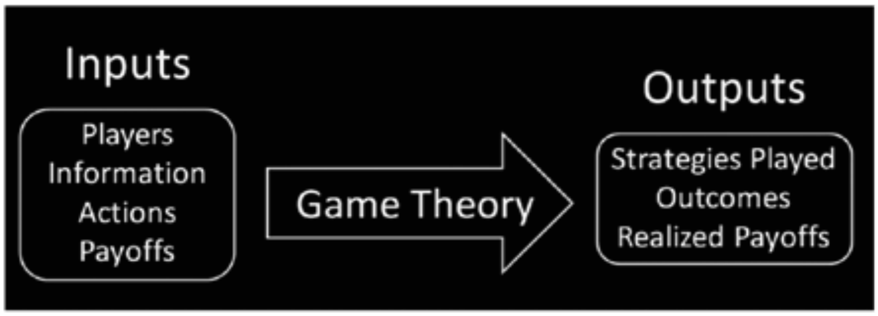


Figure 1

This is where game theory comes to play. By analyzing the decision making of producers and consumers, and abstracting them into game theoretical models, we can accurately predict how it can impact the environment through the different strategies they may choose.

There commonly involves multiple scenarios in environmental sustainability which parallel classic game theory examples. One of them is the prisoner's dilemma, which highlights the failure of cooperation with the introduction of a common good leading to a situation known as tragedy of the commons. We will also discuss the tendency of firms to cooperate.

The tragedy of the commons

A common good is a good that is rivalrous and yet non-excludable. It is a good which one can prevent others from having by consuming it, however, it is impossible to prevent people from accessing it. Examples include clean air and water, water can be tapped beyond replenishment while clean air is combusted in many industrial processes. A central problem is the tragedy of commons where common goods often become unsustainable due to overuse.

We first find a simple scenario to help illustrate our point, note that this is an example which can also be applied to many other common goods.

Firstly, we set the conditions for our game.

There is a total number of tuna, X, in the ocean.
We have a total of n players, each chooses to fish a certain number of tuna , let the amount each chooses to fish be $x_i \geq 0$.

The amount of tuna left will be $X - \sum_{i=1}^{n} x_i$......(1)

Each player enjoys a benefit of $ln(x_i)$, preventing others from benefiting from that amount. However, each person also benefits from the remainder of the tuna fish left, which is $ln(X - \sum_{j=1}^{n} x_j)$. This is due to the fact that the remaining tuna fish will reproduce and result in increased stock in the future.

Therefore, the total payoff for each player will be, $ln(x_i) + ln(X - \sum_{j=1}^{n} x_j)$......(2)

We find the partial derivative of the expression of the above and thus find its maximum point.

In partial differentiation, we find the derivative of one of the variables while leaving the other variables constant, in this case, the variable is x_i. The maximum is at the turning point when its first derivative is equal to 0.

$\frac{1}{x_i} + \frac{1}{X - \sum_{j=1}^{n} x_j} = 0$.

(note: remember that when differentiating, $-\sum_{j=1}^{n} x_j$ also has a negative x_i term)

Solving, we get: $x_i = \frac{X - (\sum_{j \neq i}^{n} x_j)}{2}$......(3)

Since all players will adopt the same strategy, let the amount fished by each player be x. Solving for x, we get

$x = \frac{X}{n+1}$.

Substituting this value of x with equation (2) above, we get $2nln(\frac{X}{n+1})$ in terms of total pay-off for all players combined.

It is intuitive to think that this will result in the maximum total payoff or benefit to all the players, however, is this the case?

We know that the payoff for each player is $ln(x_i) + ln(X - \sum_{j=1}^{n} x_j)$ from (2).

The expression for total combined payoff will be $\sum_{i=1}^{n} ln(x_i) + nln(X - \sum_{i=1}^{n} x_i)$......(4)

Following the procedure above, we find the partial derivative of x_i, which is

$\frac{1}{x_i} - \frac{n}{X - \sum_{j=1}^{n} x_j} = 0$

Given that all players consume the same amount, we can solve for x again,

$x = \frac{X}{2n}$. This is known as the Pareto optimality consumption, as no change can lead to a player gaining a higher payoff without another player losing some payoff.

Substituting the value of x back to (4), the maximum total combined pay off is actually

$(nln(\frac{x}{2n}) + nln(\frac{x}{2})) \geq 2nln(\frac{X}{n+1})$. The equality occurs when n=1. Dividing both sides by n, it also stands true that at pareto optimality consumption, payoff of each player will also be larger than before.

Tragedy of the commons refers to a scenario where no rational agent cooperates despite there being a strategy which would maximize collective payoff if adopted by all players. In the above example, we see how each player should consume $x = \frac{X}{2n}$ to maximize total pay-off. However, it is not true that it is the optimal strategy for the individual player, this is due to the fact that it is not a Nash equilibrium.

The Nash equilibrium refers to the scenario that no player can obtain a higher payoff by choosing an action different from its current one, given that all other players adhere to their action. A rational agent will only choose the best option among alternatives, thus, (3) is the Nash equilibrium as it is derived from the maximum turning point of x_i.

Infinite iteration of repeated prisoner's dilemma

B A	**B** **Reduce**	**B** **Do not reduce**
A **Reduce**	3 3	1 4
A **Do not reduce**	4 1	2 2

Figure 2.

Let's say that we have a game where each player can choose to either reduce or not reduce emissions. As shown in figure 2 above, the greatest total benefit derived is when both players choose to reduce emissions. However, since the atmosphere is a common good, it is impossible to prevent the other player from also benefiting from the reduced emissions, even by the other player. Given that it takes costs to reduce emissions, it will be more rational for a player to choose not to reduce, looking at figure 2, the column for reducing emissions will be (1,2) however, the column for not reducing emissions will be (3,4) .

This is called a strictly dominated strategy to reduce. Thus, the only Nash-equilibrium to not reduce emissions for both.

Of course, we see that the above situation is not realistic as in reality, the climate game is played across a much bigger stretch of time. Given that the strategy for every player is to maximize total benefits across an infinite period of time, how will the strategy change?

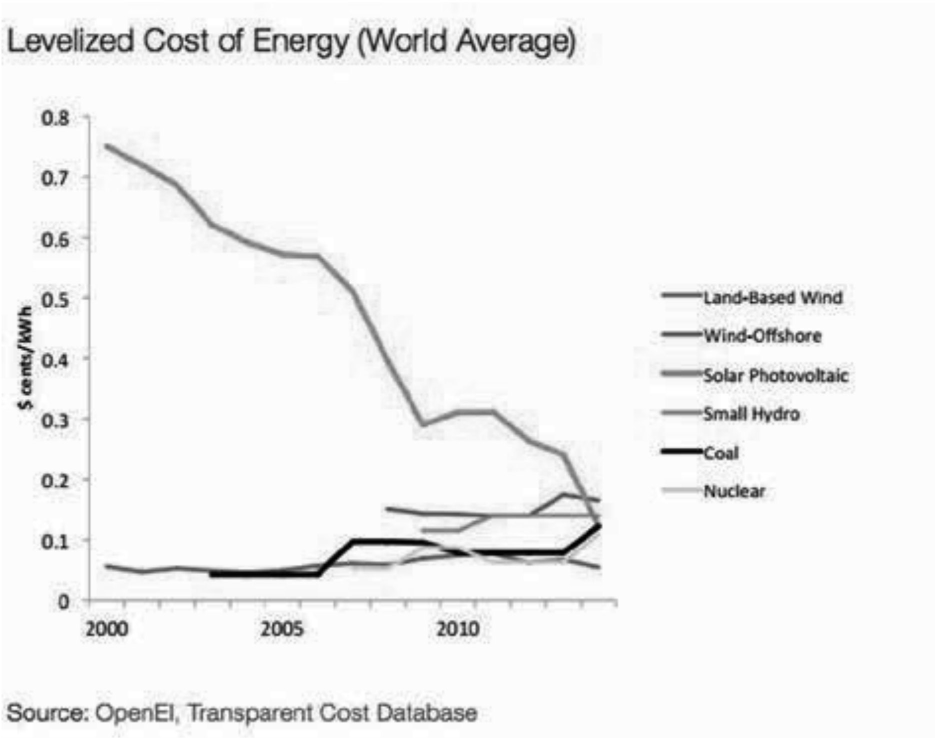
Firstly, we will calculate the total payoff for mutual cooperation assuming it goes on forever. As such, we will need to repeat the game an infinite number of times. However, notice that the payoff for each game will not be the same as players would rather receive the payoff on an earlier game compared to a later one. This will require the use of a discount function, $0 \leqslant \delta \leq 1$. The payoff for each game decreases by a fixed percentage for each subsequent game.

Payoff for first game: 3
Payoff for second game: 3δ

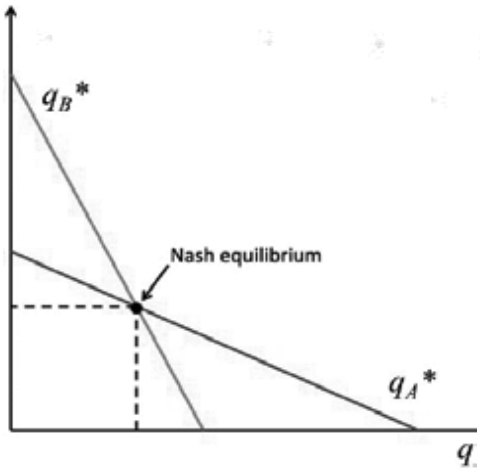
For total payoff, it is an geometric series whereby $a = 3$ and common ratio $r = \delta$. Since $0 \leqslant \delta \leq 1$, the sum to infinity of the series is convergent.

Calculating, we get total sum $S = \frac{3}{1-\delta}$. Clearly, this is larger than the total sum $S = \frac{2}{1-\delta}$ if players do not reduce emissions for all games.

In real life examples, firms and consumers may also face the same conundrum of whether to switch to renewable sources of energy given that it usually results in higher costs. Although the repercussions and costs incurred from not addressing environmental sustainability will be even greater.

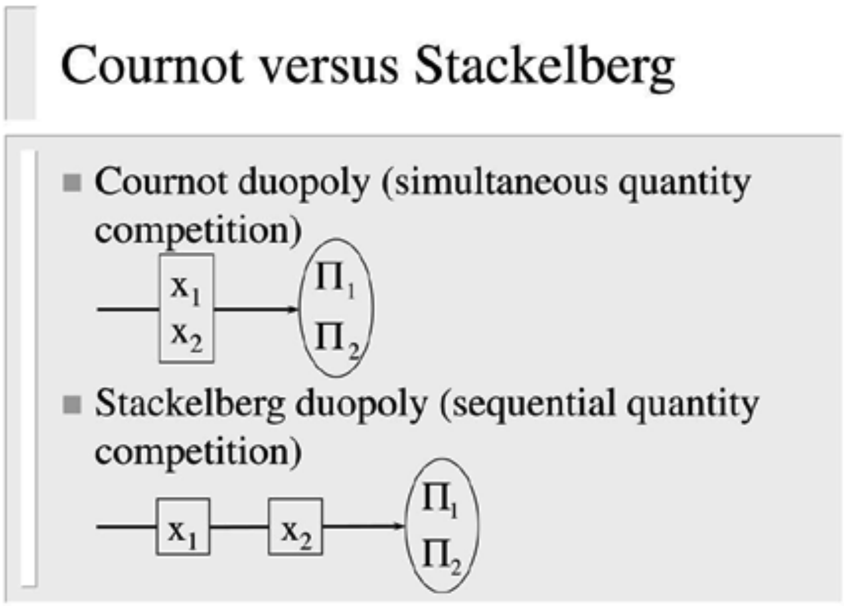


By investing more into renewable energy by both firms and consumers, the total payoff increases as research and development causes costs of renewable energy to decrease further.



We have considered the tragedy of the commons assuming that all players are equal and all players fish simultaneously. This sort of game is also called a symmetrical game, the conclusion is that all players will have the same outcome at Nash equilibrium, as shown in the graph above.

In the case where the players are firms in a market, this is known as cournot competition where firms compete for output to increase total revenue which is given by the product of price and quantity.



In stackelberg competition, the game is not symmetrical as there are firms which enter the industry slower than others, these are known as followers. Followers are able to make decisions based on the action of industry leaders, or firms that have entered the industry before them. Thus, the output and profits of each firm also differs.

One thing which changes is that following firms may benefit from the research and development of the earlier firm, such as research on green R&D. This is illustrated in the extended form diagram below.

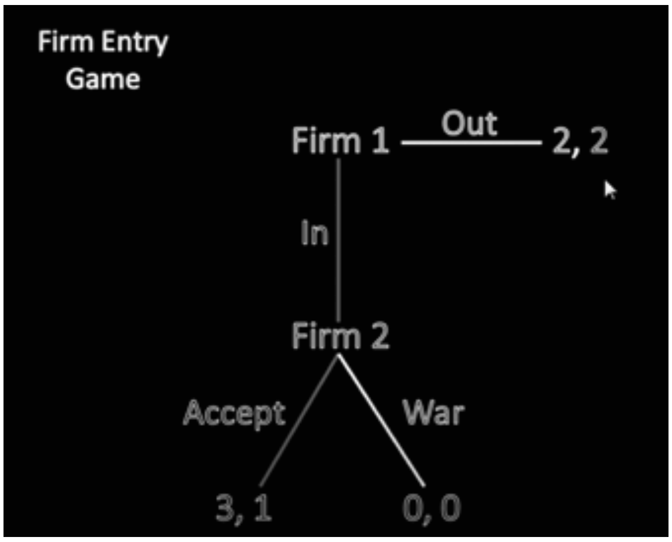


Figure 3.

We will use a method called backwards induction to show why this is the case. Backwards induction involves looking at the final outcome and predicting what action each firm will take based on the final outcome.

Firm 1 starts the game by choosing either in or out. Let's say that he chooses to go in. Now it is firm 2's turn, he can choose to either declare war or accept, as a rational agent, he will choose accept given that he earns a greater payoff 1 compared to zero if he declares war. In this scenario, firm 1 gains a payoff of 3, larger than the payoff of 1 if he chooses the alternative of going out.

Conclusion

We have examined several tools that help us understand sustainability in game theory as maximizing total pay-offs across multiple games. Specifically, we are able to show that common goods should be regulated to increase total payoff. It is thus in the best interests of the government and other organizations to subsidize research on sustainability given that more firms will benefit and subsequently reduce their emissions by sharing their research results.

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