USING A MODEL OF DIFFERENTIAL EQUATIONS TO ASSESS THE EFFECTIVENESS OF PUBLIC HEALTH MEASURES TO MITIGATE THE SPREAD OF COVID-19

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ABSTRACT

In the face of a pandemic, public health measures are essential in slowing the spread of the disease and getting it under control to prevent healthcare facilities from being overwhelmed. However, with the wide range of measures, it can be difficult to choose which to implement. Mathematical modelling provides a statistical method of answering similar questions, allowing the relationships between measures and statistics to be clearly visualised. As such, this essay aims to use the data from Singapore's response to the COVID-19 pandemic, and plot a series of differential equations to see the change in the basic reproduction number, R_0, of the virus over time. With the change in R_0 being related to when the measures were implemented, their effectiveness in decreasing the spread of COVID-19 can be compared and utilised when addressing future pandemics.

SECTION 1: INTRODUCTION

Over the course of the COVID-19 pandemic, the rate of transmission of the virus has varied due to the various public health measures that have been put into effect such as masks being mandatory, or the implementation of the phase system. The basic reproductive number of a virus, R_0, is equal to the number of people each infected individual infects each day, and is essential when studying and modelling pandemics. While standard Susceptible-Infected-Recovered (SIR) models assume R_0, to be constant, public health measures change R_0 in reality. This paper will propose a method to quantitatively assess the effectiveness of the aforementioned healthcare measures in mitigating the spread of COVID-19 by modelling and analysing how R_0 varies as safety measures are implemented or relaxed by the government.

R_0 at any time during the pandemic will be obtained by modelling existing data on COVID-19 using the equations of an SIR model:

$$\frac{ds}{dt} = -\beta s(t)i(t) \text{ ....(1)}$$

$$\frac{di}{dt} = \beta s(t)i(t) - \gamma \text{ ....(2)}$$

$$\frac{dr}{dt} = \gamma i(t) \text{........................(3)}$$

Where t is the number of days after the start of the pandemic, $s(t)$, $i(t)$ and $r(t)$ are the susceptible, infected and recovered fraction of the population respectively, while β and γ are the infection rate and the recovery rate. γ refers to the fraction of the infected population who recover each day, and will be analysed as a constant for the purposes of the model. β measures the number of people that an infected individual potentially spreads the disease to per day [1].

Through determining R_0 at any given time, the effectiveness of healthcare measures can be estimated based on the change in R_0 after the implementation of said measures, providing insight into the efficacy of measures for reference when tackling future pandemics.

SECTION 2: MODEL

2.1. Assumptions

No.	Assumptions	Rationale
1	Upon recovery, the patient cannot become reinfected	Recovered patients have antibodies to fight off the covid-19 virus and hence have an immunity against the virus.
2	The death of an infected person counts as a recovery	The infected count is based on those with currently detected and active cases of COVID-19, while the recovered number indicates those who had COVID-19 in the past and can no longer be infected. As such, as a dead infected person is no longer an active case and cannot become reinfected, they are counted as a recovery
3	The infected and susceptible individuals are mixed equally throughout society	With clusters of infected or susceptible people, the spread of the disease would not be impacted by the current number of infected
4	The recovery rate, γ, is constant $\frac{1}{14}$	A patient takes maximum 14 days to recover [2], so we assume that $\frac{1}{14}$of infected recover each day.

Additionally, since the model aims to provide a retrospective analysis of safety measures, only the period between the start of the pandemic, 15th February 2020, and 15th August 2020 ($t = 0$ to $t = 183$) were considered.

2.2. Developing the Model

The approach taken for developing the model was to work backwards from real-world data to find the equations for an SIR model accurate to the COVID-19 pandemic in Singapore. Since R_0 is given by $\frac{\beta}{\gamma}$[3] and γ is assumed to be constant, an expression of β in terms of t must first be found.

Equation (2) was used to obtain the expression of βsince it accounts for γ.

By manipulating (2),

$$\beta = \frac{1}{s(t) \cdot i(t)} [\frac{di}{dt} + \gamma i(t)]$$

$$\beta = \frac{1}{s(t)} [\frac{\frac{di}{dt}}{i(t)} + \frac{1}{14}] \ldots\ldots\ldots\ldots\ldots..(3)$$

Hence, to find β for any given t, $\frac{di}{dt}$, $s(t)$ and $i(t)$ need to be found.

2.2.1 Finding $i(t)$ and $\frac{di}{dt}$

A table of number of infected over time was made [4], then the fraction of infected over time ($i(t)$) was found by dividing the number of infected by 5,850,000, the approximate population of Singapore [5].

t / days	No. of infected	i(t) x 10^{-3}	t / days	No. of infected	i(t) x 10^{-3}	t / days	No. of infected	i(t) x 10^{-3}	t / days	No. of infected	i(t) x 10^{-3}	t / days	No. of infected	i(t) x 10^{-3}	t / days	No. of infected	i(t) x 10^{-3}
0	54	0.0094737	32	196	0.034386	64	5809	1.0191	96	17672	3.1004	128	6697	1.1749	160	4176	0.73263
1	56	0.0098246	33	221	0.038772	65	7202	1.2635	97	17408	3.0540	129	6411	1.1247	161	4509	0.79105
2	53	0.0092982	34	254	0.044561	66	8275	1.4518	98	17163	3.0111	130	6298	1.1049	162	4821	0.84579
3	52	0.0091228	35	290	0.050877	67	9233	1.6198	99	16717	2.9328	131	6106	1.0712	163	5119	0.89807
4	50	0.0087719	36	309	0.054211	68	10242	1.7968	100	16199	2.8419	132	6104	1.0709	164	5277	0.92579
5	48	0.0084211	37	355	0.062281	69	11107	1.9486	101	15876	2.7853	133	6057	1.0626	165	5406	0.94842
6	39	0.0068421	38	400	0.070175	70	11679	2.0489	102	15577	2.7328	134	5925	1.0395	166	5474	0.96035
7	40	0.0070175	39	469	0.082281	71	12552	2.2021	103	14932	2.6196	135	5650	0.99123	167	5687	0.99772
8	38	0.0066667	40	509	0.089298	72	13314	2.3358	104	14206	2.4923	136	5381	0.94404	168	5745	1.0079
9	41	0.0071930	41	547	0.095965	73	13809	2.4226	105	13616	2.3888	137	5085	0.89211	169	5872	1.0302
10	33	0.0057895	42	602	0.10561	74	14439	2.5332	106	13162	2.3091	138	4855	0.85175	170	5845	1.0254
11	31	0.0054386	43	629	0.11035	75	14910	2.6158	107	12802	2.2460	139	4684	0.82175	171	5865	1.0289
12	30	0.0052632	44	648	0.11368	76	15817	2.7749	108	12637	2.2170	140	4521	0.79316	172	6459	1.1332
13	29	0.0050877	45	683	0.11982	77	16184	2.8393	109	12799	2.2454	141	4333	0.76018	173	6497	1.1398
14	30	0.0052632	46	752	0.13193	78	16779	2.9437	110	12994	2.2796	142	4240	0.74386	174	6458	1.1330
15	32	0.0056140	47	779	0.13667	79	17303	3.0356	111	12950	2.2719	143	4111	0.72123	175	6319	1.1086
16	30	0.0052632	48	827	0.14509	80	17873	3.1356	112	12943	2.2707	144	3948	0.69263	176	6162	1.0811
17	32	0.0056140	49	886	0.15544	81	18544	3.2533	113	12999	2.2805	145	3751	0.65807	177	5656	0.99228
18	33	0.0057895	50	983	0.17246	82	19207	3.3696	114	12903	2.2637	146	3807	0.66789	178	5198	0.91193
19	36	0.0063158	51	1025	0.17982	83	19647	3.4468	115	12612	2.2126	147	3731	0.65456	179	4848	0.85053
20	48	0.0084211	52	1098	0.19263	84	20144	3.5340	116	12408	2.1768	148	3650	0.64035	180	4734	0.83053
21	48	0.0084211	53	1211	0.21246	85	20595	3.6132	117	12076	2.1186	149	3716	0.65193	181	4504	0.79018
22	60	0.010526	54	1444	0.25333	86	20541	3.6037	118	11785	2.0675	150	3865	0.67807	182	4113	0.72158
23	67	0.011754	55	1609	0.28228	87	20799	3.6489	119	11363	1.9935	151	3863	0.67772	183	3767	0.66088
24	73	0.012807	56	1763	0.30930	88	20516	3.5993	120	10989	1.9279	152	3843	0.67421			
25	82	0.014386	57	1964	0.34456	89	20104	3.5270	121	10426	1.8291	153	3849	0.67526			
26	91	0.015965	58	2323	0.40754	90	19622	3.4425	122	9780	1.7158	154	3795	0.66579			
27	103	0.018070	59	2631	0.46158	91	18992	3.3319	123	9252	1.6232	155	3799	0.66649			
28	107	0.018772	60	3037	0.53281	92	18676	3.2765	124	8735	1.5325	156	3637	0.63807			
29	121	0.021228	61	3734	0.65509	93	18486	3.2432	125	8130	1.4263	157	3823	0.67070			
30	134	0.023509	62	4331	0.75982	94	18407	3.2293	126	7583	1.3304	158	3922	0.68807			
31	152	0.026667	63	5241	0.91947	95	18135	3.1816	127	7127	1.2504	159	4056	0.71158			

Table 1. Table of number and fraction of infected population over time.

$i(t) \times 10^{3}$ is tabulated instead of $i(t)$ for presentation purposes. A scatterplot of $i(t)$ was plotted and divided into sections at the local minimum points. Local minimums were chosen as partitions to allow accurate approximation of each section since each wave of a pandemic roughly follows normal distribution[6]. This allows best-fit Gaussian curves[7] plotted in LoggerPro to accurately model $i(t)$. The domains of the 3 sections obtained are $0 \leq t \leq 108$, $108 < t \leq 156$ and $156 < t \leq 183$. The small wave present at $148 < t \leq 156$ was not considered as a separate section since it was relatively small compared to the other waves, and was considered as part of the second section. The best-fit Gaussian curves of each section of the scatter plot provided the equation for $i(t)$ in each domain.

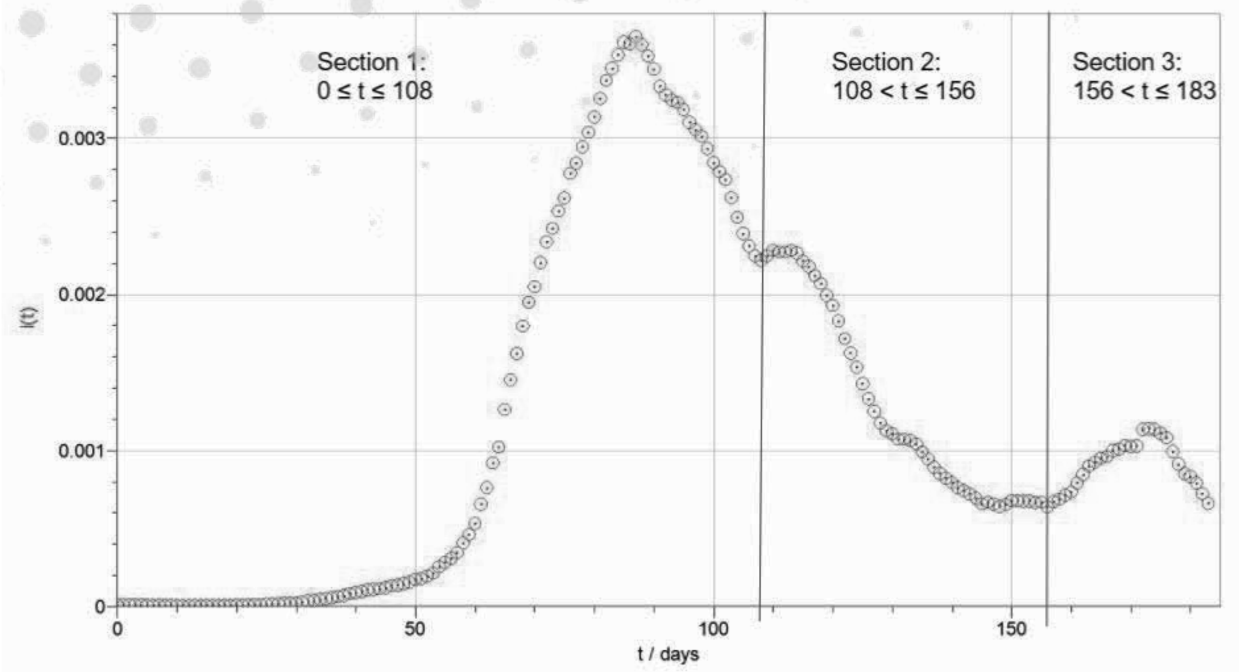

Figure 1. Scatter plot of active COVID-19 cases over time split into 3 sections.

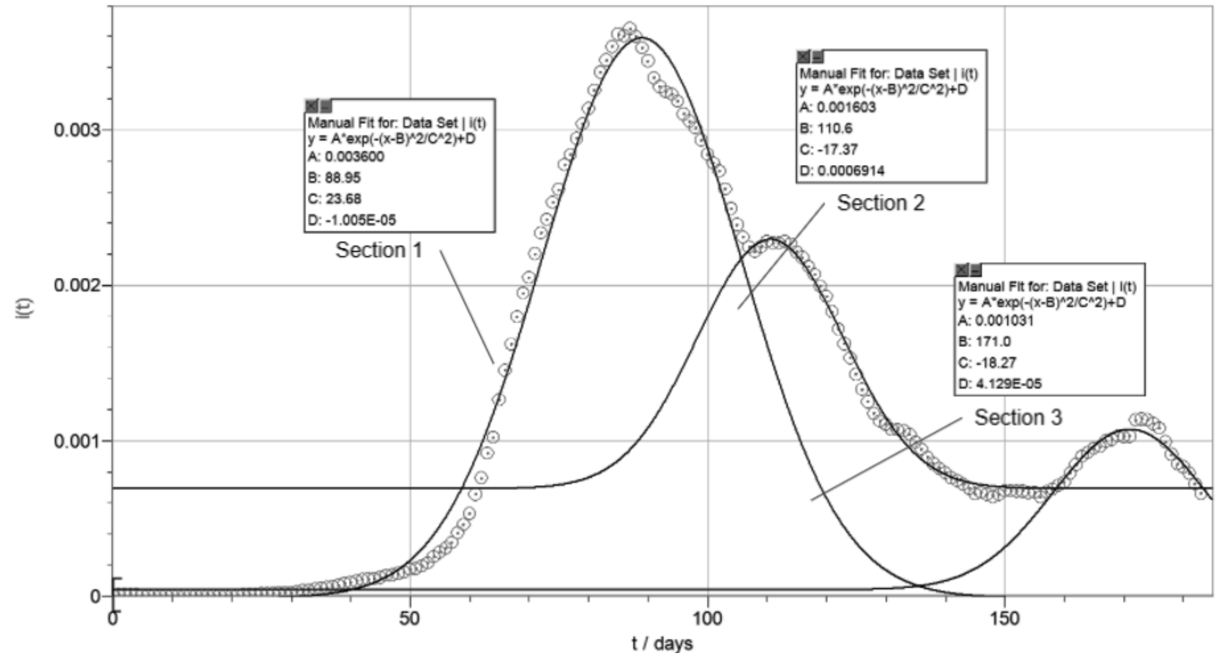

Figure 2. Scatter plot of active COVID-19 cases over time modelled using 3 Gaussian curves.

From the graph, the split function for $i(t)$ was obtained:

$$i(t) := \begin{cases} (3.6 \times 10^{-3} e^{\left(\frac{-(t-88.95)^2}{560.74}\right)} - 1.005 \times 10^{-5}, & for\ 0 < t \leq 108 \\ (1.603 \times 10^{-3}) e^{\left(\frac{-(t-110.6)^2}{301.71}\right)} + 6.914, & for\ 108 < t \leq 156 \\ (1.031 \times 10^{-3}) e^{\left(\frac{-(t-171)^2}{339.79}\right)} + 4.129 \times 10^{-5}, & for\ 156 < t \leq 183 \end{cases} \quad \text{...... (4)}$$

Using the derivative calculator from Wolfram Alpha, the differentiated form of the split function was obtained:

$$\frac{di}{dt} := \begin{cases} -e^{\left(\frac{-(t-88.95)^2}{560.74}\right)}(1.2840 \times 10^{-5} \times t - 1.1421 \times 10^{-3}), & for < t \leq 108 \\ -e^{\left(\frac{-(t-110.6)^2}{301.71}\right)}(6.6288 \times 10^{-3} \times t - 7.3315), & for\ 108 < t \leq\ 156 \\ -e^{\left(\frac{-(t-171)^2}{339.79}\right)}(1.0564 \times 10^{-3} \times t - 6.1775 \times 10^{-6}), & for\ 156 < t \leq 183 \end{cases} \quad \text{...... (5)}$$

2.2.2. Finding $s(t)$

$s(t)$ can be expressed in terms of $i(t)$ and $r(t)$ using the principle that the sum of fractions of infected, recovered and susceptible is equal to one, the total population of Singapore.

$$i(t) + r(t) + s(t) = 1$$
$$s(t) = 1 - i(t) - r(t) \text{............(6)}$$

Thus, $r(t)$ must be found. The method used to find $i(t)$ was used to find $r(t)$ and $\frac{dr}{dt}$. Since data on the fraction of recovered over time was not available, a dataset for this metric was extrapolated using the data on infected over time, newly infected per day and the assumptions of the model. The total number of recovered over time is equal to the sum of the number of recovered per day, so one way to create the necessary dataset is to find the dataset of fraction of recovered per day and sum them for each day. To find these values, the following relationship was used:

$$I(t) = I(t-1) + I_d(t-1) - R_d(t-1)$$
$$R_d(t-1) = I(t-1) + I_d(t-1) - I(t) \text{............(7)}$$

Where $I_d(t)$ denotes number of newly infected in day t

and $R_d(t)$ denotes number of newly recovered in day t

Data on the number of newly infected per day (Worldometers, n.d.) was tabulated into table 2, then substituted into (6) along with data from table 1 to find the dataset of $r(t)$.

t / days	No. of newly infected	t / days	No. of newly infected	t / days	No. of newly infected	t / days	No. of newly infected	t / days	No. of newly infected	t / days	No. of newly infected
0	3	32	32	64	1426	96	614	128	119	160	513
1	2	33	40	65	1111	97	642	129	191	161	481
2	4	34	47	66	1016	98	548	130	113	162	469
3	3	35	23	67	1037	99	344	131	219	163	359
4	1	36	54	68	897	100	383	132	291	164	334
5	1	37	49	69	618	101	533	133	213	165	278
6	3	38	73	70	931	102	373	134	202	166	396
7	0	39	52	71	799	103	611	135	246	167	307
8	1	40	49	72	528	104	506	136	215	168	313
9	1	41	70	73	690	105	518	137	188	169	226
10	2	42	42	74	528	106	408	138	169	170	295
11	3	43	35	75	932	107	544	139	185	171	908
12	2	44	47	76	447	108	569	140	136	172	301
13	4	45	74	77	657	109	517	141	183	173	242
14	4	46	49	78	573	110	261	142	156	174	132
15	2	47	65	79	632	111	344	143	158	175	175
16	2	48	75	80	788	112	383	144	125	176	188
17	2	49	120	81	741	113	386	145	191	177	61
18	5	50	66	82	768	114	218	146	170	178	42
19	13	51	106	83	753	115	451	147	178	179	102
20	8	52	142	84	876	116	422	148	322	180	83
21	12	53	287	85	451	117	463	149	346	181	81
22	10	54	198	86	884	118	347	150	249	182	86
23	6	55	191	87	675	119	407	151	248	183	91
24	12	56	233	88	752	120	214	152	327		
25	9	57	386	89	793	121	151	153	202		
26	13	58	334	90	465	122	247	154	257		
27	12	59	447	91	682	123	257	155	123		
28	14	60	728	92	305	124	142	156	399		
29	17	61	623	93	451	125	218	157	310		
30	23	62	942	94	570	126	262	158	354		
31	47	63	596	95	448	127	218	159	277		

Table 2. Table of number of newly infected individuals per day.

t / days	No. of recovered	Total no. recovered	r(t) x 10³	t / days	No. of recovered	Total no. recovered	r(t) x 10³	t / days	No. of recovered	Total no. recovered	r(t) x 10³
0	1	1	0.00017544	32	7	106	0.018596	64	33	794	0.13930
1	5	6	0.0010526	33	7	113	0.019825	65	38	832	0.14596
2	5	11	0.0019298	34	11	124	0.021754	66	58	890	0.15614
3	5	16	0.0028070	35	4	128	0.022456	67	28	918	0.16105
4	3	19	0.0033333	36	8	136	0.023860	68	32	950	0.16667
5	10	29	0.0050877	37	4	140	0.024561	69	46	996	0.17474
6	2	31	0.0054386	38	4	144	0.025263	70	58	1054	0.18491
7	2	33	0.0057895	39	12	156	0.027368	71	37	1091	0.19140
8	-2	31	0.0054386	40	11	167	0.029298	72	33	1124	0.19719
9	9	40	0.0070175	41	15	182	0.031930	73	60	1184	0.20772
10	4	44	0.0077193	42	15	197	0.034561	74	57	1241	0.21772
11	4	48	0.0084211	43	16	213	0.037368	75	25	1266	0.22211
12	3	51	0.0089474	44	12	225	0.039474	76	80	1346	0.23614
13	3	54	0.0094737	45	5	230	0.040351	77	62	1408	0.24702
14	2	56	0.0098246	46	22	252	0.044211	78	49	1457	0.25561
15	4	60	0.0105263	47	17	269	0.047193	79	62	1519	0.26649
16	0	60	0.0105263	48	16	285	0.050000	80	117	1636	0.28702
17	1	61	0.0107018	49	23	308	0.054035	81	78	1714	0.30070
18	2	63	0.0110526	50	24	332	0.058246	82	328	2042	0.35825
19	1	64	0.0112281	51	33	365	0.064035	83	256	2298	0.40316
20	8	72	0.0126316	52	29	394	0.069123	84	425	2723	0.47772
21	0	72	0.0126316	53	54	448	0.078596	85	505	3228	0.56632
22	3	75	0.013158	54	33	481	0.084386	86	626	3854	0.67614
23	0	75	0.013158	55	37	518	0.090877	87	958	4812	0.84421
24	3	78	0.013684	56	32	550	0.096491	88	1164	5976	1.0484
25	0	78	0.013684	57	27	577	0.101228	89	1275	7251	1.2721
26	1	79	0.013860	58	26	603	0.105789	90	1095	8346	1.4642
27	8	87	0.015263	59	41	644	0.112982	91	998	9344	1.6393
28	0	87	0.015263	60	31	675	0.118421	92	495	9839	1.7261
29	4	91	0.015965	61	26	701	0.122982	93	530	10369	1.8191
30	5	96	0.016842	62	32	733	0.128596	94	842	11211	1.9668
31	3	99	0.017368	63	28	761	0.133509	95	911	12122	2.1267

t / days	No. of recovered	Total no. recovered	r(t) x 10³	t / days	No. of recovered	Total no. recovered	r(t) x 10³	t / days	No. of recovered	Total no. recovered	r(t) x 10³
96	878	13000	2.2807	128	405	36003	6.3163	160	180	45361	7.9581
97	887	13887	2.4363	129	304	36307	6.3696	161	169	45530	7.9877
98	994	14881	2.6107	130	305	36612	6.4232	162	171	45701	8.0177
99	862	15743	2.7619	131	221	36833	6.4619	163	201	45902	8.0530
100	706	16449	2.8858	132	338	37171	6.5212	164	205	46107	8.0889
101	832	17281	3.0318	133	345	37516	6.5818	165	210	46317	8.1258
102	1018	18299	3.2104	134	477	37993	6.6654	166	183	46500	8.1579
103	1337	19636	3.4449	135	515	38508	6.7558	167	249	46749	8.2016
104	1096	20732	3.6372	136	511	39019	6.8454	168	186	46935	8.2342
105	972	21704	3.8077	137	418	39437	6.9188	169	253	47188	8.2786
106	768	22472	3.9425	138	340	39777	6.9784	170	275	47463	8.3268
107	709	23181	4.0668	139	348	40125	7.0395	171	314	47777	8.3819
108	407	23588	4.1382	140	324	40449	7.0963	172	263	48040	8.4281
109	322	23910	4.1947	141	276	40725	7.1447	173	281	48321	8.4774
110	305	24215	4.2482	142	285	41010	7.1947	174	271	48592	8.5249
111	351	24566	4.3098	143	321	41331	7.2511	175	332	48924	8.5832
112	327	24893	4.3672	144	322	41653	7.3075	176	694	49618	8.7049
113	482	25375	4.4518	145	135	41788	7.3312	177	519	50137	8.7960
114	509	25884	4.5411	146	246	42034	7.3744	178	392	50529	8.8647
115	655	26539	4.6560	147	256	42290	7.4193	179	216	50745	8.9026
116	754	27293	4.7882	148	259	42549	7.4647	180	313	51058	8.9575
117	754	28047	4.9205	149	197	42746	7.4993	181	472	51530	9.0404
118	769	28816	5.0554	150	251	42997	7.5433	182	432	51962	9.1161
119	781	29597	5.1925	151	268	43265	7.5904	183	-	-	-
120	777	30374	5.3288	152	321	43586	7.6467				
121	797	31171	5.4686	153	256	43842	7.6916				
122	775	31946	5.6046	154	253	44095	7.7360				
123	774	32720	5.7404	155	285	44380	7.7860				
124	747	33467	5.8714	156	213	44593	7.8233				
125	765	34232	6.0056	157	211	44804	7.8604				
126	718	34950	6.1316	158	220	45024	7.8989				
127	648	35598	6.2453	159	157	45181	7.9265				

Table 3. Table of data on recovered individuals.

$r(t) \times 10^{3}$ is tabulated instead of $r(t)$ for presentation purposes. Although data for day 183 is missing, the same approach was taken to find $r(t)$ as $i(t)$ since the loss of one data point is unlikely to cause significant inaccuracy.

From Table 3, a scatter plot of fraction of recovered over time was plotted and split into sections spanning the same domains as in the fraction of infected. This is to ensure that the values of R_0 produced by the model are accurate.

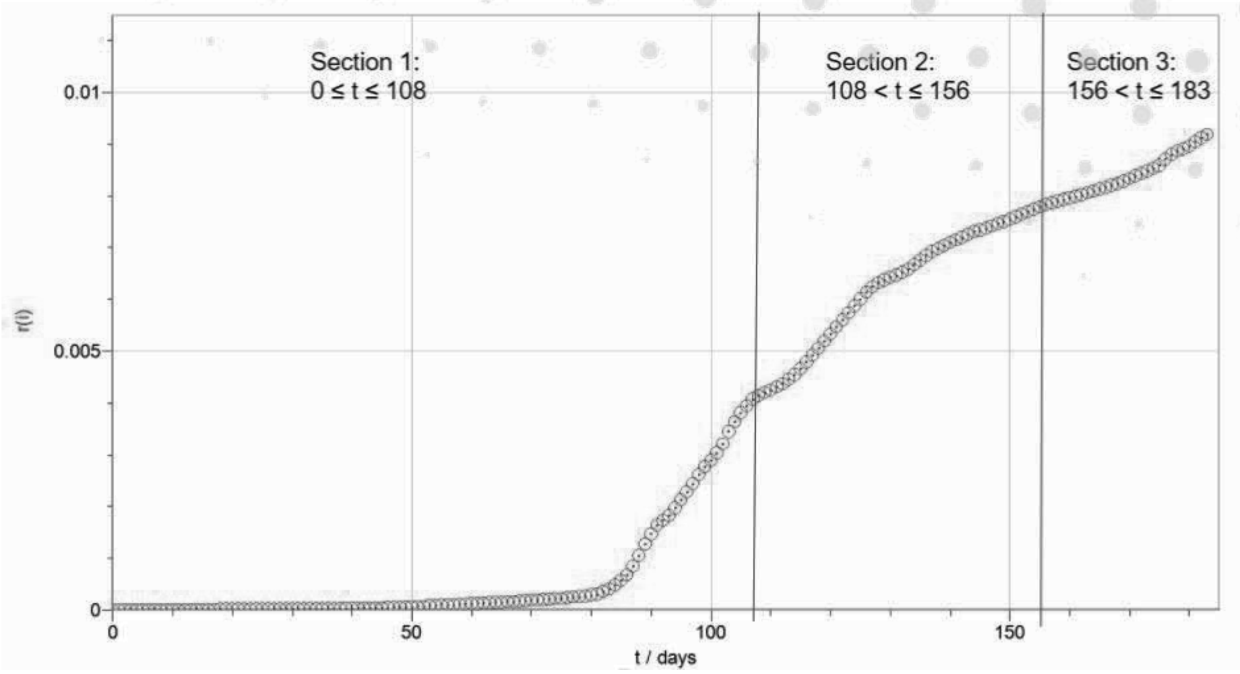

Figure 3. Scatter plot of fraction recovered over time.

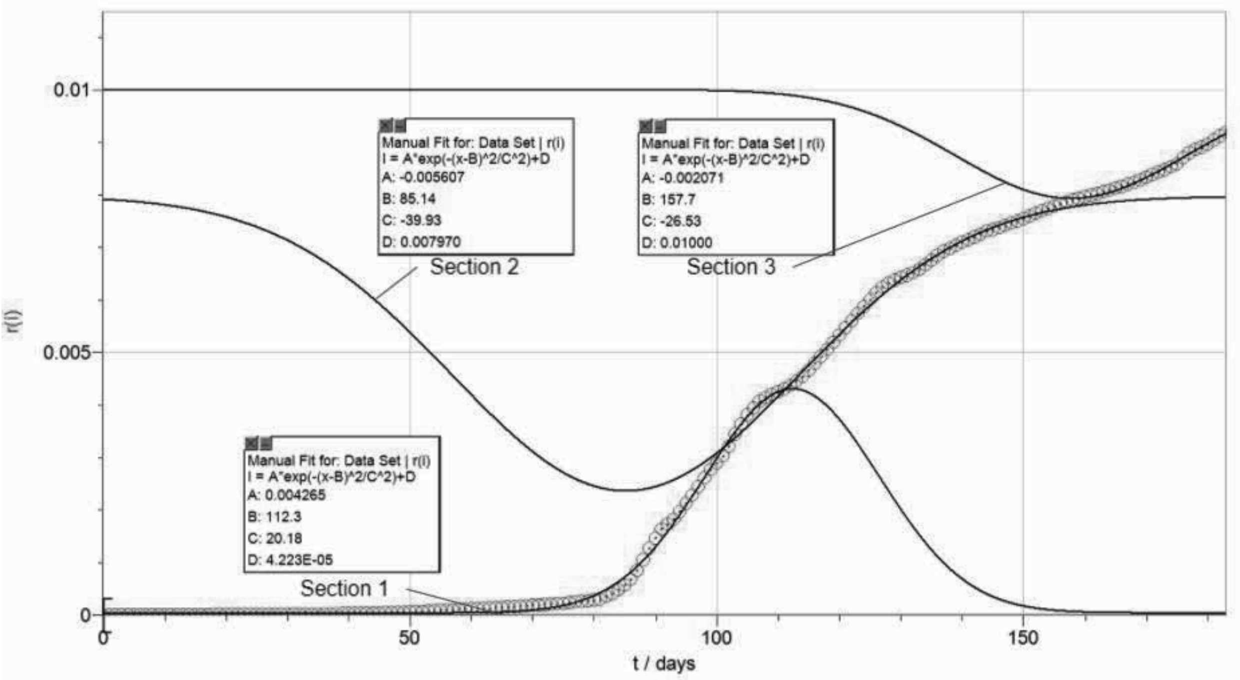

Figure 4. Scatter plot of fraction recovered over time modeled using 3 Gaussian curves.

Like with $i(t)$, Gaussian curves were used to approximate a split function for $r(t)$, then differentiated to obtain $\frac{dr}{dt}$.

From the graph,

$$r(t) := \begin{cases} (4.265 \times 10^{-3}) e^{\left(\frac{-(t-112.3)^2}{407.23}\right)} + 4.233 \times 10^{-5}, & for\, 0 < t \leq 108 \\ (-5.607 \times 10^{-3}) e^{\left(\frac{-(t-85.14)^2}{1594.4}\right)} + 7.97 \times 10^{-3}, & for\, 87 < t \leq 156 \\ (-2.071 \times 10^{-3}) e^{\left(\frac{-(t-157.7)^2}{703.84}\right)} + 1.00 \times 10^{-2}, & for\, 156 < t \leq 183 \end{cases} \quad \text{......} \ (8)$$

Using Wolfram Alpha derivative calculator,

$$\frac{dr}{dt} := \begin{cases} e^{\left(\frac{-(t-112.3)^2}{407.23}\right)} (2.0946 \times 10^{-5} \times t - 2.3522 \times 10^{-3}, & for\, 0 < t \leq 108 \\ e^{\left(\frac{-(t-85.14)^2}{1594.4}\right)} (7.0333 \times 10^{-6} \times t - 5.9882 \times 10^{-4}, & for\, 87 < t \leq 156 \\ e^{\left(\frac{-(t-157.7)^2}{703.84}\right)} (5.8848 \times 10^{-3} \times t - 0.92804, & for\, 156 < t \leq 183 \end{cases} \quad \text{......} \ (9)$$

Once the equations for $i(t)$, $r(t)$, $\frac{di}{dt}$ and $\frac{dr}{dt}$ are obtained, $s(t)$ can be found.

By substituting (4) and (8) into (6),

$$s(t) := \begin{cases} -(3.6 \times 10^{-3}) e^{\left(\frac{-(t-88.95)^2}{560.74}\right)} - (4.625 \times 10^{-3}) e^{\left(\frac{-(t-112.3)^2}{407.23}\right)} + 0.99996772, & for\, 0 < t \leq 108 \\ (5.607 \times 10^{-3}) e^{\left(\frac{-(t-85.14)^2}{1594.4}\right)} - (1.625 \times 10^{-3}) e^{\left(\frac{-(t-110.6)^2}{301.71}\right)} + 0.9913386, & for\, 108 < t \leq 156 \\ (2.071 \times 10^{-3}) e^{\left(\frac{-(t-157.7)^2}{703.84}\right)} - (1.031 \times 10^{-3}) e^{\left(\frac{-(t-171)^2}{339.79}\right)} + 0.98995871, & for\, 156 < t \leq 183 \end{cases} \quad \text{...} \ (10)$$

2.2.3. Finding β and R_0

After obtaining $s(t)$, the split function for β can be found and plotted.

By substituting (4), (5), and (9) into (2),

$$\beta := \begin{cases} \dfrac{\dfrac{-e^{\left(\frac{-(t-88.95)^2}{560.74}\right)}(1.2840 \times 10^{-5} \times t - 1.1421 \times 10^{-3})}{-(3.6 \times 10^{-3}) e^{\left(\frac{-(t-88.95)^2}{560.74}\right)} - 1.005 \times 10^{-5}} + \gamma}{-(3.6 \times 10^{-3}) e^{\left(\frac{-(t-88.95)^2}{560.74}\right)} - (4.625 \times 10^{-3}) e^{\left(\frac{-(t-112.3)^2}{407.23}\right)} + 0.99996772}, & for\ 0 < t \leq 108 \\ \dfrac{\dfrac{-e^{\left(\frac{-(t-110.6)^2}{301.71}\right)}(6.6288 \times 10^{-3} \times t - 7.3315)}{(1.625 \times 10^{-3}) e^{\left(\frac{-(t-110.6)^2}{301.71}\right)} - 6.718 \times 10^{-4}} + \gamma}{(5.607 \times 10^{-3}) e^{\left(\frac{-(t-85.14)^2}{1594.4}\right)} - (1.625 \times 10^{-3}) e^{\left(\frac{-(t-110.6)^2}{301.71}\right)} + 0.9913386}, & for\ 108 < t \leq 156 \\ \dfrac{\dfrac{-e^{\left(\frac{-(t-171)^2}{339.79}\right)}(1.0564 \times 10^{-3} \times t - 6.1775 \times 10^{-6})}{0.001031 e^{\left(\frac{-(t-171)^2}{339.79}\right)} - 4.129 \times 10^{-5}} + \gamma}{(2.071 \times 10^{-3}) e^{\left(\frac{-(t-157.7)^2}{703.84}\right)} - (1.031 \times 10^{-3}) e^{\left(\frac{-(t-171)^2}{339.79}\right)} + 0.98995871}, & for\ 156 < t \leq 183 \end{cases}$$

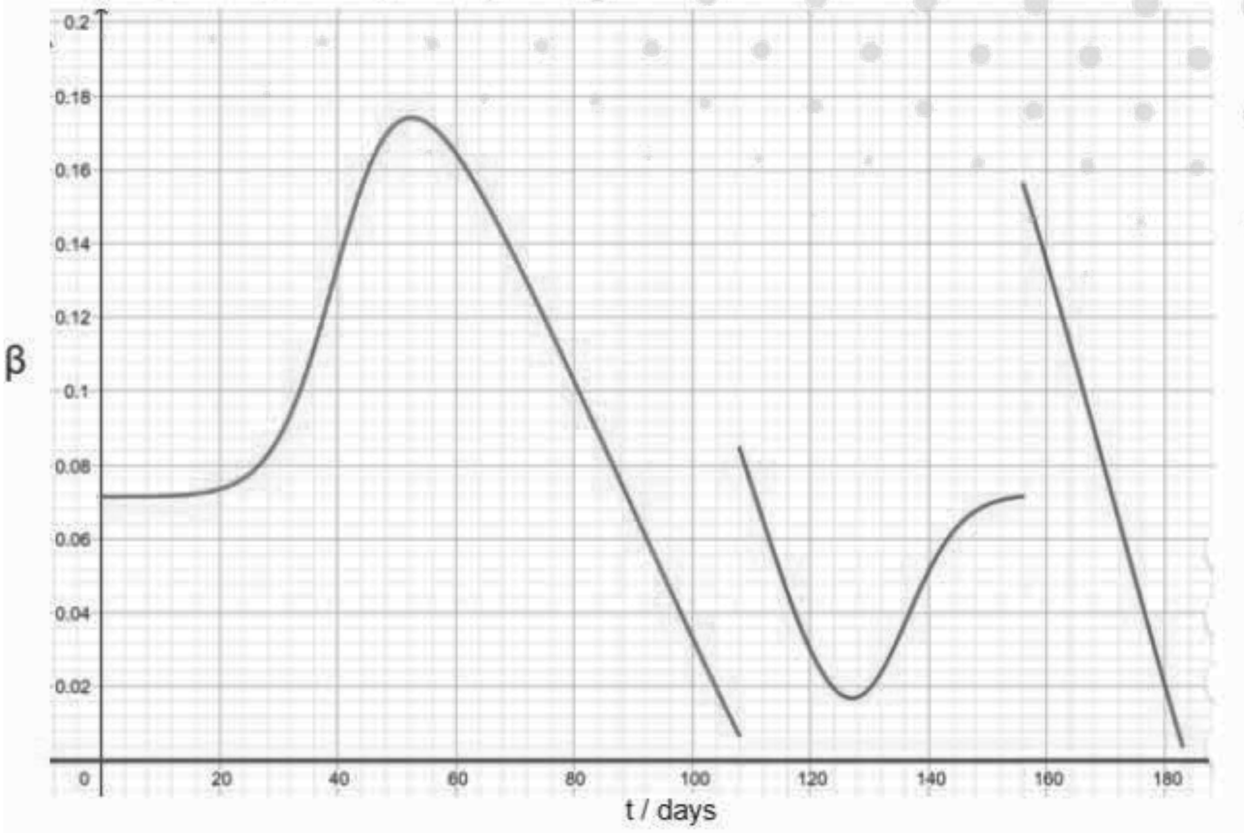

Figure 5. Split function of β over t.

Finally, using the split function of β, R_0 at any point in time can be found by dividing β by γ, $\frac{1}{14}$.

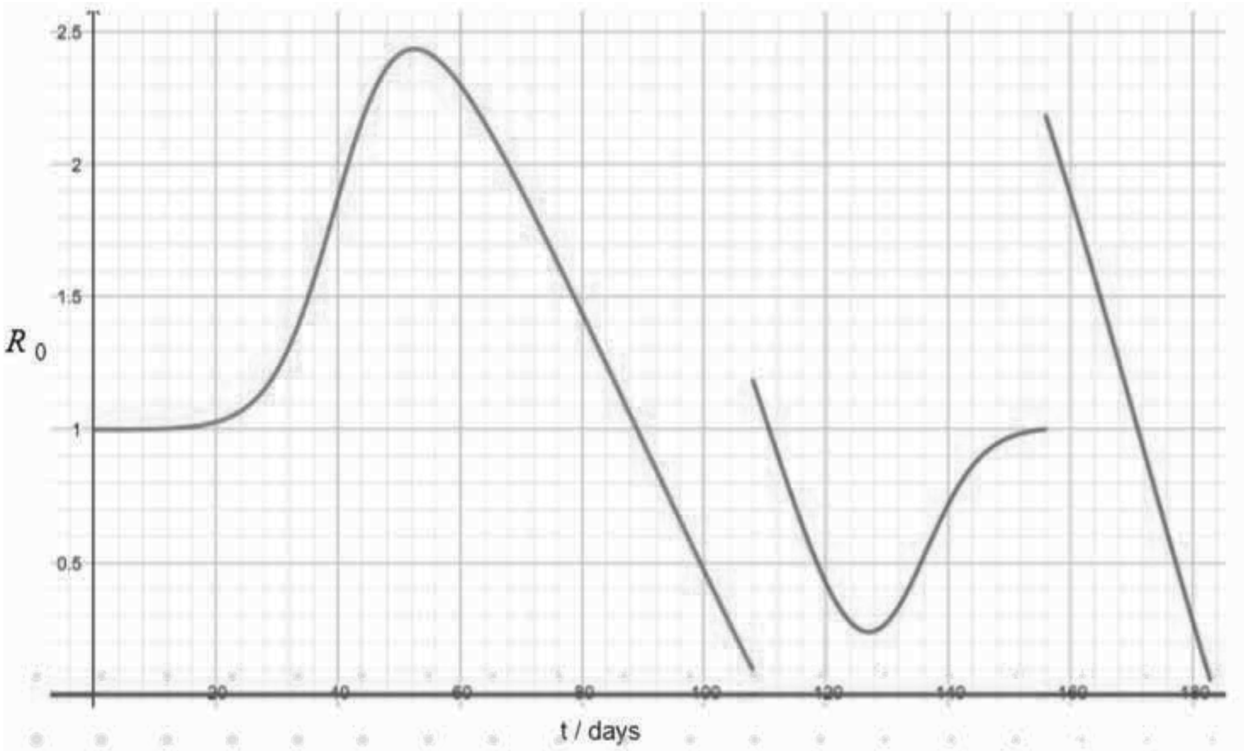

Figure 6. Split function of R_0 over t.

SECTION 3: DISCUSSING THE MODEL

To discuss the relationship between R_0 and safety measures, significant changes to safety measures and their date of implementation were tabulated.

No.	Change to safety measure	Date of implementation	Day of implementation (in terms of t)
1	Safe distancing measures	March 20th	34
2	Circuit breaker	April 7th	52
3	Stadiums closed	April 10th	55
4	Compulsory mask-wearing	April 14th	59
5	Circuit breaker extended, number of essential businesses reduced, entry restrictions in wet markets and supermarkets	April 21st	66
6	Restrictions eased - essential businesses reopened	May 5th	80
7	Circuit breaker ended	June 1st	107
8	Phase 2	June 19th	125

Table 4. Table of changes to safety measures and their date of implementation

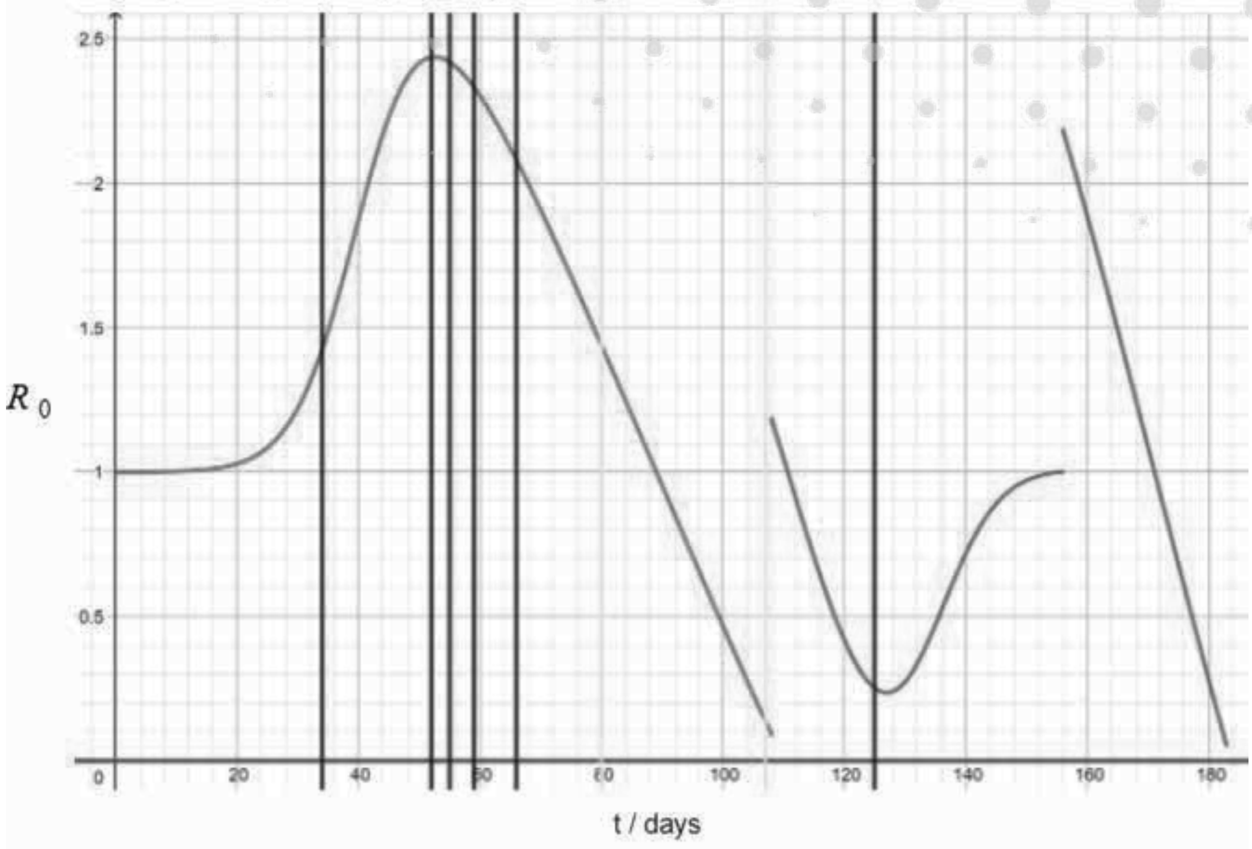

Figure 7. Split function of R_0 over t with black lines to show times of safety measure implementation and yellow lines to show times of safety measure removals.

The first safety measure was implemented at t = 34, as R_0 is increasing. The maximum R_0 at 2.44 occurs on 7th April (t = 52), and the circuit breaker is implemented on the same date. Compulsory mask-wearing was also implemented on April 14th. The effectiveness of the circuit breaker and compulsory mask-wearing as a healthcare measure can be seen clearly as R_0 decreased by 2414% from t = 52 to t = 107, indicating that the virus will not be able to continue its propagation amongst susceptible individuals since $R_0 < 1$ at 0.0927.

However, immediately upon the ending of circuit breaker at t = 107, when R_0 is at its minimum of 0.0927, R_0 rapidly increases. This suggests that the removal of circuit breaker could have been delayed or made more gradual to avoid the another wave of the pandemic which is indicated by the spike in R_0.

Following this, R_0 reaches a second minimum at 0.236 when t = 127. No safety measure can be directly attributed to this change, so it is possible that the surge in R_0 and the resultant spike in $i(t)$ caused the population to reduce their contact with others independent of safety measures.

The effect of ending circuit breaker can be seen again from t = 127 to t = 148, as R_0 reaches 2.184, indicating that the pandemic will worsen if no measures are taken since $R_0 > 1$. This supports the conclusion that the circuit breaker was removed too early. Subsequently, R_0 starts to steadily decrease until t = 183 to 0.05, which is likely a result of implementing Phase 2 measures at t = 125. This proves the effectiveness of phase 2 measures in mitigating the spread of COVID-19, though a better management strategy would be to implement it earlier. The drastic increase in R_0 from t = 148 to t = 149 is likely an error of the model and will be discussed in its limitations.

SECTION 4: EVALUATION OF MODEL

4.1 Limitations

One limitation of the model is that it does not allow comparisons of healthcare measures in isolation, causing ambiguity when evaluating individual safety measures since changes in R_0 are caused by multiple measures. Furthermore, our model only covered the first 183 days and did not expand to include other measures implemented later.

The use of split functions to model R_0 introduces inaccuracy in interpreting the model. R_0 is unable to vary as drastically as indicated at the upper and lower limit of each domain in the split function, potentially causing values of R_0 near these regions to be inaccurate.

4.2 Advantages

Typical SIR models assume R_0 to be constant, but this is rarely the case in reality. Our model allows for a more dynamic situation, where the infectivity of the disease can vary over time, providing a more realistic approach. Additionally, the model allows for the study of R_0 as opposed to $i(t)$, allowing for direct analysis of infectivity rates, the quantity that safety measures aim to reduce.

The use of real world data and the partitioning of the model makes it more accurate than conventional SIR models which only provide rough approximations of a pandemic's course because they cannot represent complex changes, such as the presence of multiple maximum and minimum points in $i(t)$.

4.3 Conclusion

Through evaluating the model, it can be concluded that circuit breaker was the most effective measure in managing the pandemic, as it caused the largest decrease in R_0. However, the closeness at which the measures were implemented resulted in difficulties knowing which measure was the most impactful. However, the trend was that measures aimed at reducing contact between the population were the most effective. In order to obtain more accurate results, it would be necessary to compare the measures in isolation, but this would likely be unethical in reality. As such, constructing a simulation based on the contact and infection rates would be the next step towards obtaining the best measure.

SECTION 5: REFERENCES

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