Climate Change: A Case of The Tragedy of the Commons And Its Potential Soution

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Abstract

The world is currently undergoing rapid climate change and temperatures could rise by $3-5$ degrees by 2100 [1], bringing about severe repercussions and making the issue of climate change extremely relevant to our lives. In this essay, we will be illustrating the depletion of natural resources and by extension, climate change, as a case of the tragedy of the commons using game theory and account for the lack of progress in climate change negotiations between countries using the aforementioned framework and introduce potential solution thereafter.

Introduction

In the context of our essay, the tragedy of the commons refers to countries overconsuming fossil fuels in order to maximise self benefit, causing the depletion of fossil fuels and catalysing climate change, bringing about detriments to all. For the purpose of our essay, we will take consumption of fossil fuels as the main source of greenhouse gases that leads to climate change.

The n-countries Tragedy of the Commons

There are n countries choosing how much fossil fuel they want to consume.

Suppose there are K units of fossil fuels in the environment where consumption will be from a common resource pool shared by the n countries.

Each country, x_i, consumes k_i of fossil fuels for its production purposes, where $k_i \geq 0,\ \forall i = 1, 2, \ldots n$. Hence, the remaining amount of fossil fuels is given by

$$K - \sum_{i=1}^{n} k_i.$$

Suppose the benefit of consuming k_i amount of fossil fuels is equivalent to $\ln(k_i)$. Thus, the benefit that each country enjoys from the remaining amount of fossil fuels is given by

$$\ln\left(K - \sum_{i=1}^{n} k_i\right).$$

By letting the total benefit (payoff) of country x_i be ν_i, the payoff of country x_i, which takes into account its own action, k_i, and other countries' action, k_{-i}, is given by

$$\nu_i(k_i) = \ln(k_i) + \ln\left(K - \sum_{j=1}^{n} k_j\right).$$

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In order to maximise benefit, the country will choose the best-response where the first-order condition holds, i.e. $\dfrac{\partial \nu_i}{\partial k_i} = 0$. Hence,

$$\begin{aligned}
\frac{\partial \nu_i(k_i)}{\partial k_i} &= 0 \\
\frac{\partial}{\partial k_i}\left[\ln(k_i) + \ln\left(K - \sum_{j=1}^{n} k_j\right)\right] &= 0 \\
\frac{1}{k_i} + \frac{1}{K - \sum_{j=1}^{n} k_j} \cdot \frac{\partial}{\partial k_i}[-k_i] &= 0 \qquad \text{, Since } k_i \text{ occurs somewhere in } \sum_{j=1}^{n} k_j \\
\frac{1}{k_i} - \frac{1}{K - \sum_{j=1}^{n} k_j} &= 0 \\
k_i &= K - \sum_{j=1}^{n} k_j \\
2k_i &= K - \sum_{j \neq i} k_j \qquad \text{, Since } k_i \text{ occurs somewhere in } \sum_{j=1}^{n} k_j \\
k_i &= \frac{1}{2}\left(K - \sum_{j \neq i} k_j\right) \quad , \forall i = 1, 2, \ldots, n.
\end{aligned}$$

Thus, the best response function for country x_i in response to other countries, b_i, is

$$b_i = \frac{1}{2}\left(K - \sum_{j \neq i} k_j\right).$$

By considering a symmetric Nash Equilibrium, i.e. every country chooses the same level of consumption of fossil fuels, lets call it k^*, then we have,

$$\begin{aligned}
k^* &= \frac{1}{2}\left(K - \sum_{j \neq i} k^*\right) \\
2k^* &= [K - (n-1)k^*] \\
(n+1)k^* &= K \\
\therefore k^* &= \frac{K}{n+1}.
\end{aligned}$$

However, consuming $\frac{K}{n+1}$ amount of fossil fuels only maximises the country's own payoff, if all countries were to do so, we argue that this will not benefit everyone. To prove this, we look to maximise the social welfare function, $\omega(k_i)$, which is the sum of payoffs from the n countries. So,

$$\begin{aligned}\omega(k_i) &= \sum_{i=1}^{n} \nu_i(k_i) \\ &= \sum_{i=1}^{n} \ln(k_i) + n \ln\left(K - \sum_{j=1}^{n} k_j\right).\end{aligned}$$

Taking the first-order condition on ω, i.e. letting $\frac{\partial \omega(k_i)}{\partial k_i} = 0$, we have,

$$\begin{aligned}\frac{\partial \omega(k_i)}{\partial k_i} &= 0 \\ \frac{\partial}{\partial k_i}\left\{\sum_{i=1}^{n} \ln(k_i) + n\ln\left(K - \sum_{j=1}^{n} k_j\right)\right\} &= 0 \\ \frac{1}{k_i} + \frac{n}{K - \sum_{j=1}^{n} k_j} \cdot \frac{\partial}{\partial k_i}[-k_i] &= 0 \qquad \text{, Since } k_i \text{ occurs somewhere in } \sum_{j=1}^{n} k_j \\ \frac{1}{k_i} - \frac{n}{K - \sum_{j=1}^{n} k_j} &= 0 \\ nk_i &= K - \sum_{j=1}^{n} k_j \\ (n+1)k_i &= K - \sum_{j \neq i} k_j \qquad \text{, Since } k_i \text{ occurs somewhere in } \sum_{j=1}^{n} k_j \\ \therefore k_i &= \frac{1}{n+1}\left(K - \sum_{j \neq i} k_j\right) \quad , \forall i = 1, 2, \ldots, n.\end{aligned}$$

Similar to the analysis for the Nash equilibrium, the Pareto optimal consumption, the level of consumption where the total payoff of countries are maximised, should also be symmetrical, i.e every country will be consuming the same Pareto optimal consumption level, lets call it k^p, thus,

$$\begin{aligned}k^p &= \frac{1}{n+1}\left(K - \sum_{j \neq i} k^p\right) \\ (n+1)k^p &= K - (n-1)k^p \\ (n+1+n-1)k^p &= K \\ k^p &= \frac{K}{2n}.\end{aligned}$$

Hence, in order to maximise social welfare, each country should be consuming only $\frac{K}{2n}$ amount of fossil fuels but the Nash equilibrium for n countries is overconsuming fossil fuels. To see this, the total consumption of n countries at Nash equilibrium and at Pareto optimal level are $\frac{nK}{n+1}$ and $\frac{K}{2}$ respectively. Observe that,

$$\begin{aligned}\frac{nK}{n+1} &> \frac{nK}{n+n} \\ &= \frac{K}{2}.\end{aligned}$$

For $n \geq 2$, we have $\frac{nK}{n+1} > \frac{K}{2}$, and the claim is justified. Since the countries consume $\frac{nK}{n+1}$ amount of fossil fuel to maximise its own payoff, with the above claim, all countries would suffer detrimental effects of climate change, resulting in the tragedy of the commons.

The 2-countries Tragedy of the Commons

Suppose now there are 2 countries, China and United State of America (USA), who consume k_1 and k_2 respectively. Using the aforementioned analysis, let the best response function of country i be $b_i(k_j)$, then the 2 best response function for USA and China are,

$$b_1(k_2) = k_1 = \frac{K - k_2}{2} \qquad \text{and} \qquad b_2(k_1) = k_2 = \frac{K - k_1}{2}.$$

Solving simultaneously,

$$\begin{aligned}
k_2 &= \frac{K - \frac{K-k_2}{2}}{2} \qquad \text{, Substituting } k_1 = \frac{K-k_2}{2} \text{ into } k_2 = \frac{K-k_1}{2} \\
2k_2 &= K - \frac{K-k_2}{2} \\
4k_2 &= 2K - (K - k_2) \\
\therefore k_2 &= \frac{K}{3} \\
\Rightarrow k_1 &= \frac{K - \frac{K}{3}}{2} \\
k_1 &= \frac{K}{3}.
\end{aligned}$$

Hence, the Nash equilibrium for USA and China is $\frac{K}{3}$. On the other hand, the Pareto optimal consumption can be found by maximising the social welfare function of USA and China, ω. So we have,

$$k_1 = \frac{K - k_2}{3} \qquad \text{and} \qquad k_2 = \frac{K - k_1}{3}.$$

Solving simultaneously,

$$\begin{aligned}
k_1 &= \frac{K - \frac{K-k_1}{3}}{3} \qquad \text{, Substituting } k_2 = \frac{K-k_1}{3} \text{ into } k_1 = \frac{K-k_2}{3} \\
3k_1 &= K - \frac{K-k_1}{3} \\
9k_1 &= 3K - (K - k_1) \\
\therefore k_1 &= \frac{K}{4} \\
\Rightarrow k_2 &= \frac{K - \frac{K}{4}}{3} \\
k_2 &= \frac{K}{4}.
\end{aligned}$$

Hence, the Pareto optimal level of consumption is $\frac{K}{4}$.

Now, consider a climate negotiation game where China and the USA are facing the choice to either mitigate climate change by reducing greenhouse gas emissions or not. The choice to mitigate by one of the countries would benefit both, but the country taking the action would benefit less as it consumes less fossil fuel.

In order to analyse the strategies that both country will use, we will use a payoff matrix. The respective payoff is calculated based on the payoff function and the strategy that the country decides to use. The payoff will be represented as (ν_1, ν_2) where ν_1 will denote China's payoff and ν_2 will denote USA's payoff. As calculated above, reducing consumption will correspond to the Pareto optimal consumption, i.e. $k_1 = k_2 = \frac{K}{4}$, while not reducing consumption will correspond to the Nash equilibrium consumption, i.e. $k_1 = k_2 = \frac{K}{3}$. We shall now illustrate how the payoff for both countries are calculated for the four different scenarios.

When both countries reduce consumption,

$$\begin{aligned}\nu_1 = \nu_2 &= \ln \frac{K}{4} + \ln K - \frac{K}{4} - \frac{K}{4} \\ &= \ln \frac{K}{4} + \ln \frac{K}{2} \\ &= \ln \left(\frac{K^2}{8} \right).\end{aligned}$$

When one countries reduces, say China, while the other does not,

$$\begin{aligned}\nu_1 &= \ln \frac{K}{4} + \ln K - \frac{K}{4} - \frac{K}{3} \\ &= \ln \frac{K}{4} + \ln \frac{5K}{12} \\ &= \ln \left(\frac{5K^2}{48} \right) \\ \nu_2 &= \ln \frac{K}{3} + \ln K - \frac{K}{3} - \frac{K}{4} \\ &= \ln \frac{K}{3} + \ln \frac{5K}{12} \\ &= \ln \left(\frac{5K^2}{36} \right).\end{aligned}$$

When both countries choose not to reduce consumption,

$$\begin{aligned}\nu_1 = \nu_2 &= \ln \frac{K}{3} + \ln K - \frac{K}{3} - \frac{K}{3} \\ &= \ln \frac{K}{3} + \ln \frac{K}{3} \\ &= \ln \left(\frac{K^2}{9} \right).\end{aligned}$$

Using the payoff function for both countries for their different strategies yield the payoff matrix for consumption of fossil fuels as seen in Table 1.

		USA	
		Reduce consumption	Do not reduce consumption
China	Reduce consumption	$\left(\ln\left(\frac{K^2}{8}\right), \ln\left(\frac{K^2}{8}\right)\right)$	$\left(\ln\left(\frac{5K^2}{48}\right), \ln\left(\frac{5K^2}{36}\right)\right)$
	Do not reduce consumption	$\left(\ln\left(\frac{5K^2}{36}\right), \ln\left(\frac{5K^2}{48}\right)\right)$	$\left(\ln\left(\frac{K^2}{9}\right), \ln\left(\frac{K^2}{9}\right)\right)$

Table 1: Payoff matrix for consumption of fossil fuels

From the perspective of China, since $\ln\left(\frac{5K^2}{36}\right) > \ln\left(\frac{K^2}{8}\right)$ and $\ln\left(\frac{K^2}{9}\right) > \ln\left(\frac{5K^2}{48}\right)$, regardless of USA's

From the perspective of China, since $\ln\left(\frac{5K^2}{36}\right) > \ln\left(\frac{K^2}{8}\right)$ and $\ln\left(\frac{K^2}{9}\right) > \ln\left(\frac{5K^2}{48}\right)$, regardless of USA's strategies, China will have a higher payoff by not reducing consumption. Hence, China's dominant strategy will be not to reduce consumption. Similarly, USA's dominant strategy will also be to not reduce consumption.

Thus, as illustrated by Table 1, both players' dominant strategy lies in not reducing greenhouse gas emissions, leading to a payoff of $\left(\ln\left(\frac{K^2}{9}\right), \ln\left(\frac{K^2}{9}\right)\right)$.

As a result, climate negotiations fail and both countries continue to suffer under the tragedy of the commons. However, notice that $\ln\left(\frac{K^2}{8}\right) > \ln\left(\frac{K^2}{9}\right)$. Hence, this is a form of Prisoners' Dilemma because both countries would be better off should they have cooperated and reduced emissions. Therefore, to overcome this Prisoners' Dilemma, we need to alter the agents' payoff matrix to allow their dominant strategy to be combating climate change. Thus, we propose providing members with financial incentives, b, as shown in Table 2.

		USA	
		Reduce consumption	Do not reduce consumption
China	Reduce consumption	$\left(\ln\left(\frac{K^2}{8}\right)+b,\ \ln\left(\frac{K^2}{8}\right)+b\right)$	$\left(\ln\left(\frac{5K^2}{48}\right)+b,\ \ln\left(\frac{5K^2}{36}\right)\right)$
	Do not reduce consumption	$\left(\ln\left(\frac{5K^2}{36}\right),\ \ln\left(\frac{5K^2}{48}\right)+b\right)$	$\left(\ln\left(\frac{K^2}{9}\right),\ \ln\left(\frac{K^2}{9}\right)\right)$

Table 2: Payoff matrix for consumption of fossil fuels with financial incentives

In order to make both countries' payoff when reducing consumption higher, b must be greater than some value.

$$\begin{aligned}\ln\left(\frac{K^2}{8}\right)+b &> \ln\left(\frac{5K^2}{36}\right)\\ b &> \ln\left(\frac{5K^2}{36}\right)-\ln\left(\frac{K^2}{8}\right)\\ &= \ln\frac{5}{4}\end{aligned} \qquad \text{and} \qquad \begin{aligned}\ln\left(\frac{5K^2}{48}\right)+b &> \ln\left(\frac{K^2}{9}\right)\\ b &> \ln\left(\frac{K^2}{9}\right)-\ln\left(\frac{5K^2}{48}\right)\\ &= \ln\frac{48}{45}.\end{aligned}$$

Since $\ln\frac{5}{4} > \ln\frac{48}{45}$, thus the range of value of b must be $b > \ln\frac{5}{4}$, in order for the financial incentive to be effective.

Member countries should pool together funds proportionate to their nation's wealth into researching technologies to reduce greenhouse emissions and because these technologies are highly excludable, members have the opportunity to profit from it. Moreover, such an initiative would also experience network externality, as more members participate in the initiative, the greater the likelihood of it being a success.

Conclusion

Therefore, as mathematically and economically proven, incentives need to be given to countries in order to have a joint effort in reducing climate change. However, thus far, this essay has only utilised theoretical mathematics and economic approaches to the issue of climate change. In reality, countries still doubt each other on their willingness to solve the issue at hand despite mutual benefits if they cooperated, and negotiations often fail and no party takes any actions. Thus, mutual trust is imperative if countries seek to tackle climate change and this can be done through more cooperation between governments worldwide.

Ultimately, mathematics and economics have given us great insights into the problem of climate change and its potential solutions, but to properly address the issue at hand, one has to undertake a holistic approach and consider perspectives from other relevant fields such as international relations.

References

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