Addressing Inaccurate and Inappropriate Prior Knowledge

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Introduction

In my career of teaching A Level Mathematics, I have noticed that some students lacked proficiency in reasoning mathematically; even the students who have scored well for the previous level of attainment (i.e. O Levels, IGCSE or equivalent) are burdened by complicated and lengthy mathematical computations.

Test Performance does not always reflect Mathematical Understanding

Prior knowledge can hinder learning when students apply concepts to the wrong context. To develop mastery, students must acquire component skills, practice integrating them, and know when to apply what they have learned (Ambrose, Bridges, DiPietro, Lovett, & Norman, 2010).

Students can often reproduce a procedure without being able to explain the conceptual knowledge behind the steps they have taken (Ambrose, Bridges, DiPietro, Lovett, & Norman, 2010). They may even be able to match certain types of examination questions with their required 'perfect' solution, and reproduce the solution accurately and obtain distinctions for their examination. However, they may not be able to perform as well when they move from one level to the next.

Effective Strategies have "Expiry Dates"

In teaching students at A levels, I am regularly challenged with questions such as "Why can't I do this the same way as last year when I scored an A?", "My previous teacher said that for this type of question, I only need to do such-and-such. Can I not do the same?"

Some time ago, I read an article that when teachers share rules that are based on conceptual understanding or have been overgeneralized, the students may face problems later in their Mathematics learning when these rules "expire" (Karen S. Karp, 2014). The rule of adding a zero at the end of a number when multiplied by 10 will not work once decimals (or fractions) are taught; e.g. $34.5\times10\neq34.50$. Furthermore, repeated failed attempts of "expired" rules may cause learners to lower their trust in the teacher, concerned that "rules" may expire someday.

I was reminded during a professional development course that while having sufficient prior knowledge is an essential foundation for the A Level, we (including the teacher) may also be hindered by our prior knowledge (and expert blind spot) when they are inaccurate or inappropriately used. It is thus important to identify such situations and clear the misconceptions.

Why and when "rules" no longer work: Example 1

It is only ethical to explain to students why "rules" that have given them success in the past no longer work, by explaining the context which they apply it to, and avoid criticizing fellow colleagues. It will help to provide counter-examples, especially those that the students only encounter at the current level, to illustrate how the "rules" have been overgeneralized. For example, students only work with real numbers in the secondary levels and encounter complex numbers for the first time in their pre-university. The equation $x^2+1=0$ gives conflicting solutions:

Secondary	A Level / Pre-University (Chapter on Complex Numbers)
$x^2+1=0$ No solution	$x^2+1=0$ $x^2=-1$ $x=\pm\sqrt{-1}$ $=\pm i$

Both answers are correct to the respective levels of attainment, but it may be helpful in this case for teachers to be more precise in using the phrase "no REAL solution" indicating the possibility of other types of solutions available beyond the current level.

Careless use of "rules" under inappropriate conditions: Example 2

I will also emphasize that the "rules" learnt previously were not untrue: more care needs to be exercised on when the "rules" can be applied, and that their teachers probably devised the "rules" to make learning easier. This maintains consistency in Mathematics and provides a safe (psychological) learning environment.

In classroom teaching, it will help to make a conscious effort in getting students to articulate their thoughts and reason their choice of action, e.g. explain why it is reasonable to use a specific procedure learnt earlier. This will help us as educators to better gauge what our students have learnt and identify any gaps or missing prior knowledge they should already know. We can also explicitly teach the context and conditions in which a method is applicable, since research has shown that it helps to avoid applying prior knowledge to an inappropriate context.

Another common mistake involves "cancelling" the common factor on both sides of an equation when solving $\tan x = 2\sin x,\ 0 \le x \le 2\pi$:

Correct	Common Mistake
$\tan x = 2\sin x$ $\tan x - 2\sin x = 0$ $\dfrac{\sin x}{\cos x} - 2\sin x = 0$ $\sin x\left(\dfrac{1}{\cos x} - 2\right) = 0$ $\sin x = 0$ or $\dfrac{1}{\cos x} - 2 = 0$ $x = 0,\ \pi,\ 2\pi$ or $\dfrac{1}{\cos x} = 2$ $\cos x = \dfrac{1}{2}$ (1^{st} and 4^{th} quadrant) $\alpha = \dfrac{\pi}{3}$ $x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ $\therefore x = 0,\ \dfrac{\pi}{3},\ \pi,\ \dfrac{5\pi}{3},\ 2\pi$	$\tan x = 2\sin x$ $\dfrac{\sin x}{\cos x} = 2\sin x$ $\sin x\left(\dfrac{1}{\cos x}\right) = \sin x(2)$ $\dfrac{1}{\cos x} = 2$ $\cos x = \dfrac{1}{2}$ (1^{st} and 4^{th} quadrant) $\alpha = \dfrac{\pi}{3}$ $x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ [This solution does not have the set of solution when $\sin x = 0$, which is what was "cancelled out" by students]

It is important to revisit simpler examples from earlier years, not only to point out the "rule" that typically we "cancel out" non-zero constants and non-zero variables (paying extra effort to ascertain the variable involved will never be zero in all circumstances, or generally telling students to totally avoid "cancelling" any unknowns), but also to remind (or re-teach) the concept of dividing both sides by that common factor.

Anticipated Problem Areas and Expected Outcomes

With some experience in teaching the content, it is not difficult to anticipate segments where students may encounter problems with the transition to the higher level. For the example of complex numbers presented above, some scaffolding to mentally prepare the students for the transition can help with resistive emotions that hinders learning of the new material. I have also found that students often need to be reminded that old knowledge are important to retain, especially if those are the required foundation but do not receive direct rewards when mastery is displayed. For example, being able to count the number of items is a huge achievement at the kindergarten (pre-school) level, likewise for addition skills at primary levels; these are still very important for higher levels despite the attention and rewards being greatly reduced to nearly none.

Given some guidance, I expect my students to become more discerning when applying rules and shortcuts. Over time, this should be reflected in daily assignments and regular class tests where the number of conceptual errors should decrease, and the quality and accuracy of the students' responses should show increased consistency---not a random lucky guess.

Conclusion

In terms of support, I would try to diplomatically point out some common misconception and share with my department the exact area these will affect. We can also share teaching ideas as a department and view a student's Mathematics education as a full-cycle progression rather than a myopic target to complete the curriculum.

I will continue to keep track of progress through standard assignments and tests, observations from discussions as well as feedback from my learners, parents and colleagues.

References

Ambrose, S. A., Bridges, M. W., DiPietro, M., Lovett, M. C., & Norman, M. K. (2010). How Learning Works: Seven Research-Based Principles for Smart Teaching. John Wiley and Sons.

Karen S. Karp, S. B. (2014, August). 13 Rules that Expire. *Teaching Children Mathematics Volume 21 No. 1*. The National Council of Teachers of Mathematics, Inc. Retrieved from https://www.nctm.org/Publications/teaching-children-mathematics/2014/Vol21/Issue1/13-Rules-That-Expire/