58th IMO @ Rio de Janeiro, Brazil

A photo story by Ai Xinghuan, Deputy Leader of 58th IMO Singapore Team

Journey to the West

Time:
15 July, 2017, 23:10, Singapore Time

Place:
Changi Airport, Singapore

People:
Singapore's National Team for 58th International Mathematical Olympiad, comprising, from left to right, Wang Jianzhi (RI), Wang Peng Jun, Bryan (HCI), Tan Junyao, Joel (NUSH), Chew Xuan Da, Clarence (NUSH), Ng Yu Peng (HCI), Toh Shan Hong, Dylan (NUSH)

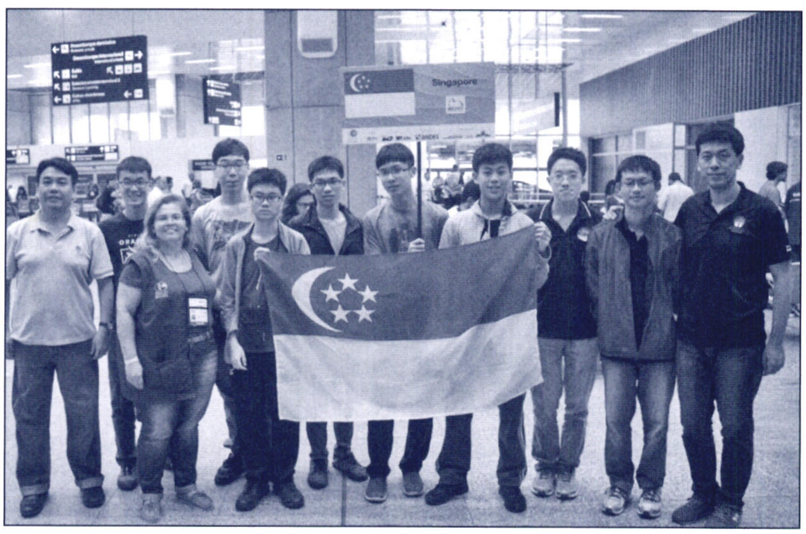
Dr Wong Yan Loi (NUS), the Team Leader, and Dr Tay Tiong Seng, Observer A, left three days earlier for Brazil to attend the jury meeting, during which the contest problems would be selected.

Observers Mr Thomas Teo Teck Kian (RI), Mr Ling Yan Hao (former contestant), and Mr Teh Jiun Harn (former member of national training team) also joined me to help the team in this journey.

Time:
16 July, 2017, 16:27, Brazil Time

Place:
International Airport of Antônio Carlos Jobim, Rio de Janeiro, Brazil

People:
Singapore's National Team with Mr Thomas Teo Teck Kian (leftmost), Claudia (local guide, third from left), Mr Teh Jiun Harn (third from right), Mr Ling Yan Hao (second from right) and me (rightmost).

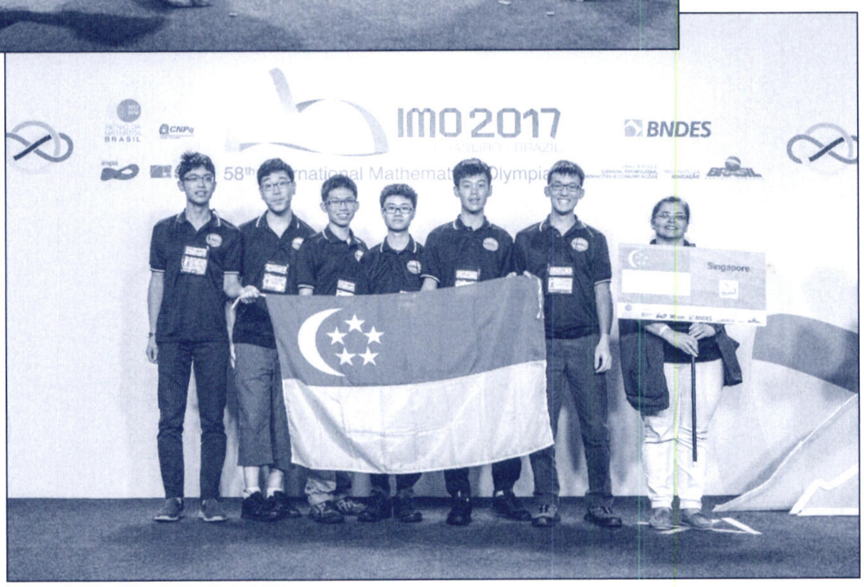


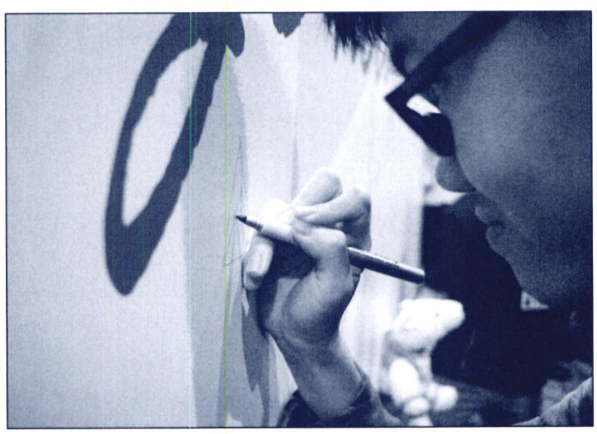
The team left Singapore in the early morning of 16 July (Singapore time), transiting at Dubai, UAE, and arrived at Rio de Janeiro, Brazil, in the afternoon of 16 July (Brazil time). The journey took about 24 hours. The organiser received us at the airport, where we met Claudia, the local guide assigned to our team, and transported us to Windsor Oceanico Hotel, Barra da Tijuca, in which the students would stay and take part in the contest. We completed registration and check-in procedures around the dinner hour. Authorisation letters for minors were required for registration.

IMO, the dream:

Time: 17 July, 2017, 14:48, Brazil Time
Place: Windsor Oceânico Hotel, Barra da Tijuca, Rio de Janeiro, Brazil

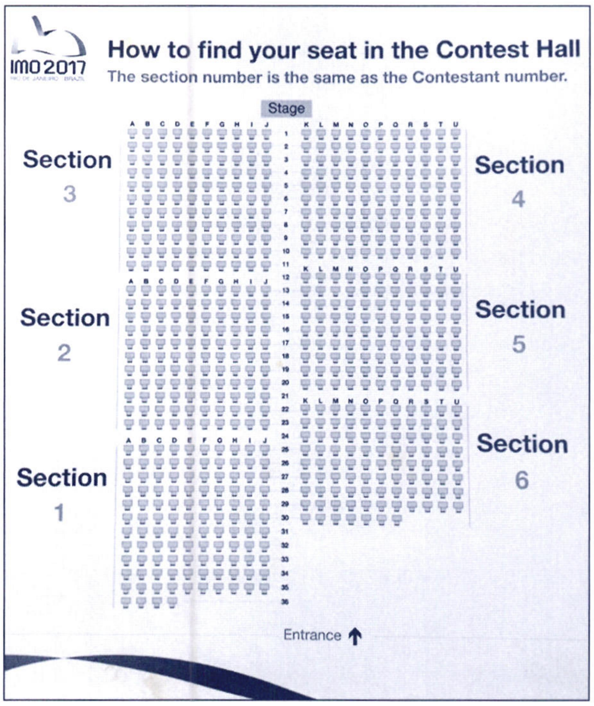
(Left) 58th IMO Mascot and Clown;
(Bottom) Singapore Team with Claudia, the local guide, at the opening ceremony





After the opening ceremony, Clarence penned down his personal message to IMO and other contestants from all over the world in mathematical language.

The team attended the opening ceremony at Windsor Oceânico Hotel. After the opening ceremony, we checked the contest venue and prepared the contestants for the first day.



Time: 19 July, 2017, 14:57, Brazil Time

Place: Windsor Oceânico Hotel, Barra da Tijuca, Rio de Janeiro, Brazil

(Top) Seating plan and the exam hall;
(Right) Jianzhi, Clarence and other contestants leaving the exam hall after finishing Day 2

The two-day contest at IMO consists of one 4.5-hour paper for each day, from 9am to 12:30pm. Each paper has three problems, covering four areas including algebra, geometry, combinatorics and number theory. The three problems on each day are usually arranged in an order of increasing difficulty (as perceived by the Jury). Each question carries seven marks. Total mark in IMO is 42.

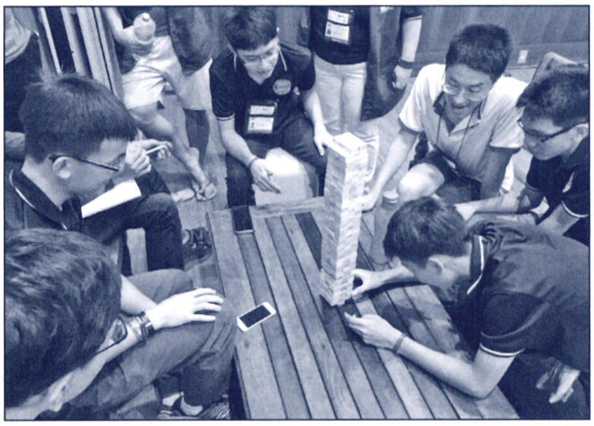
Among the three problems on the first day, we found the first one on number theory was doable, the second on algebra was difficult and the third on combinatorics was impossible for most. We met the students one by one to talk about their solutions and found that their performance matched our expectations.

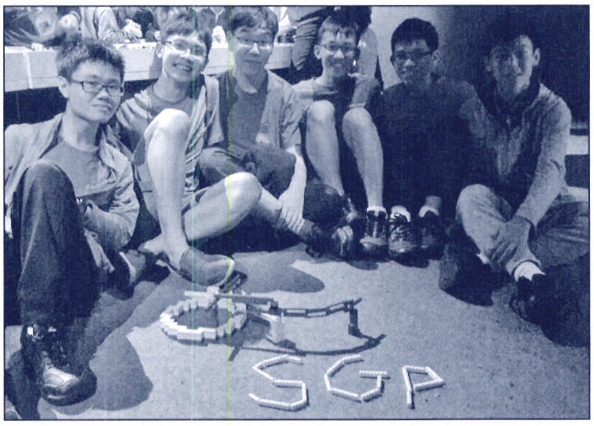
On Day 2, the students seemed to be good-spirited in general in the morning. The problems for the second day were available to the deputy leaders and observers at noon. We found that problem 4 on geometry was normal, problem 5 on combinatorics was comparable to problem 2 in difficulty, and the last problem on number theory and algebra was probably as difficult as problem 3 but partials were more possible.

From this point onwards the observer B's and I moved to another hotel to join Dr Wong and Dr Tay, who had been quarantined until then, to participate in coordination. Just before we boarded the bus for the other hotel, the students finished the second day and talked to us about their performance briefly. We decided that problem 2 and 5 were the main focus of our coordination. We expected to fight for some partials for Joel's problem 6 as well.

Journey to the West

When we worked on coordination, our students would join participants from more than one hundred countries in various activities.



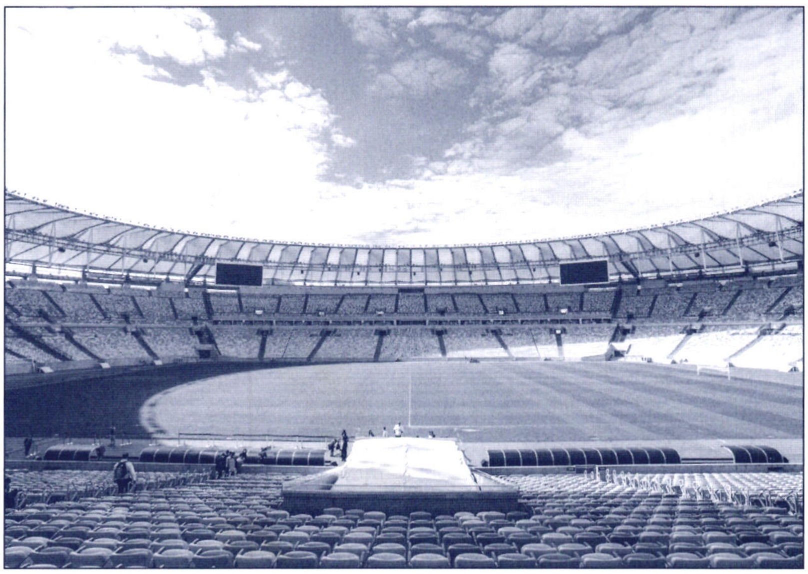


Time:
19 - 21 July, 2017, Brazil Time

Place:
Rio de Janeiro, Brazil

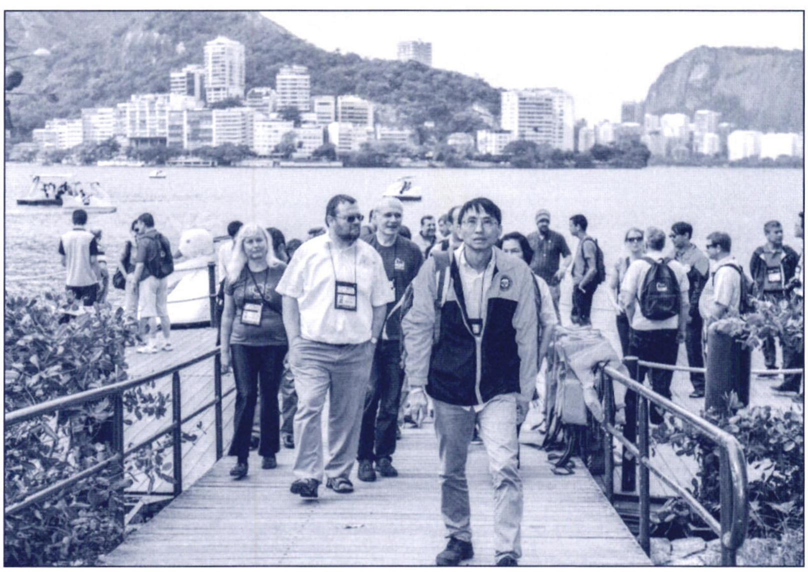
(Top Left)
Double Jenga;
(Top Right)
The art of the brick;
(Bottom)
At the beach;

(Top) Christ the Redeemer;
(Right) Maracanã Stadium



When the students were participating in the contest, team leaders and observers took a short break to discover the beauty of Rio.

(Right) Dr Wong and other teams leaders in the city tour
(Below) Dr Tay, Mr Thomas Teo and me took a walk along the beach in the early morning



Behind the Scene

Once the students started to enjoy their stay in Rio, team leaders and observers became busy. We needed to review the scripts of all our students before discussing the scoring with IMO coordinators and agreeing on the marks allocated to each solution.

Code	P1	P2	P3	P4	P5	P6	Total
RUS1	7	3	0	•	0	2	12
RUS2	7	7	•	7	0	0	21
RUS3	7	•	0	2	2	1	12
RUS4	7	7	0	7	3	•	24
RUS5	•	4	0	7	1	5	17
RUS6	7	1	0	7	•	0	15
Total	**35**	**22**	**0**	**30**	**6**	**8**	**101**

Code	P1	P2	P3	P4	P5	P6	Total
SAF1	•	1	0	2	0	0	3
SAF2	7	3	•	7	0	0	17
SAF3	7	•	0	2	3	0	12
SAF4	7	1	0	7	•	0	15
SAF5	7	0	0	•	0	0	7
SAF6	6	1	0	7	3	•	17
Total	**34**	**6**	**0**	**25**	**6**	**0**	**71**

Code	P1	P2	P3	P4	P5	P6	Total
SAU1	7	5	0	7		0	19
SAU2	•	4	0	7		0	11
SAU3	7	3	0	7		•	17
SAU4	5	•	0	1		0	6
SAU5	7	3	•	7		0	17
SAU6	7	0	0	•		0	7
Total	**33**	**15**	**0**	**29**	**0**	**0**	**77**

Code	P1	P2	P3	P4	P5	P6	Total
SGP1	7	3	0	7	•	0	17
SGP2	7	4	•	7	0	0	18
SGP3	7	4	0	•	7	4	22
SGP4	7	4	0	7	6	•	24
SGP5	7	•	0	7	7	0	21
SGP6	•	4	0	2	1	0	7
Total	**35**	**19**	**0**	**30**	**21**	**4**	**109**

Code	P1	P2	P3	P4	P5	P6	Total
SLV1	7	3	0	•	0	0	10
SLV2	7	0	0	7	0	•	14
SLV3	•	0	0	7	2	0	9
SLV4	7	2	0	4	•	0	13
Total	**21**	**5**	**0**	**18**	**2**	**0**	**46**

Code	P1	P2	P3	P4	P5	P6	Total
SRB1		•	0	7	1	0	8
SRB2		3	0	7	2	0	12
SRB3		3	•	7	0	0	10
SRB4		3	0	7	•	0	10
SRB5		7	0	•	1	0	8
SRB6		7	0	7	0	•	14
Total	**0**	**23**	**0**	**35**	**4**	**0**	**62**

Code	P1	P2	P3	P4	P5	P6	Total
SUI1	7	0	0	•		0	7
SUI2	7	0	•	4		0	11
SUI3	•	3	0	3		0	6
SUI4	7	•	0	7		0	14
SUI5	7	0	0	5		•	12
SUI6	7	3	0	2		0	12
Total	**35**	**6**	**0**	**21**	**0**	**0**	**62**

Code	P1	P2	P3	P4	P5	P6	Total
SVK1	7	•	0	2	1	0	10
SVK2	7	1	0	7	2	•	17
SVK3	7	0	0	•	1	0	8
SVK4	5	0	•	7	1	0	13
SVK5	7	0	0	7	•	0	14
SVK6	•	0	0	2	0	0	2
Total	**33**	**1**	**0**	**25**	**5**	**0**	**64**

Code	P1	P2	P3	P4	P5	P6	Total
SVN1	7	3	•	7	0	0	17
SVN2	•	1	0	7	0	0	8
SVN3	7	3	0	7	•	0	17
SVN4	7	4	0	•	0	0	11
SVN5	7	•	0	3	0	0	10
SVN6	7	0	0	7	0	•	14
Total	**35**	**11**	**0**	**31**	**0**	**0**	**77**

Code	P1	P2	P3	P4	P5	P6	Total
SWE1	7	3	0		0	0	10
SWE2	7	•	0		2	0	9
SWE3	7	1	0		•	0	8
SWE4	7	0	0		2	•	9
SWE5	•	3	0		1	0	4
SWE6	7	3	•		1	1	12
Total	**35**	**10**	**0**	**0**	**6**	**1**	**52**

Code	P1	P2	P3	P4	P5	P6	Total
SYR1	•	1	0	7		0	8
SYR2	7	7	0	7		•	21
SYR3	7	0	0	7		0	14
SYR4	0	•	0	7		0	7
SYR5	7	1	•	7		0	15
SYR6	5	1	0	•		0	6
Total	**26**	**10**	**0**	**35**	**0**	**0**	**71**

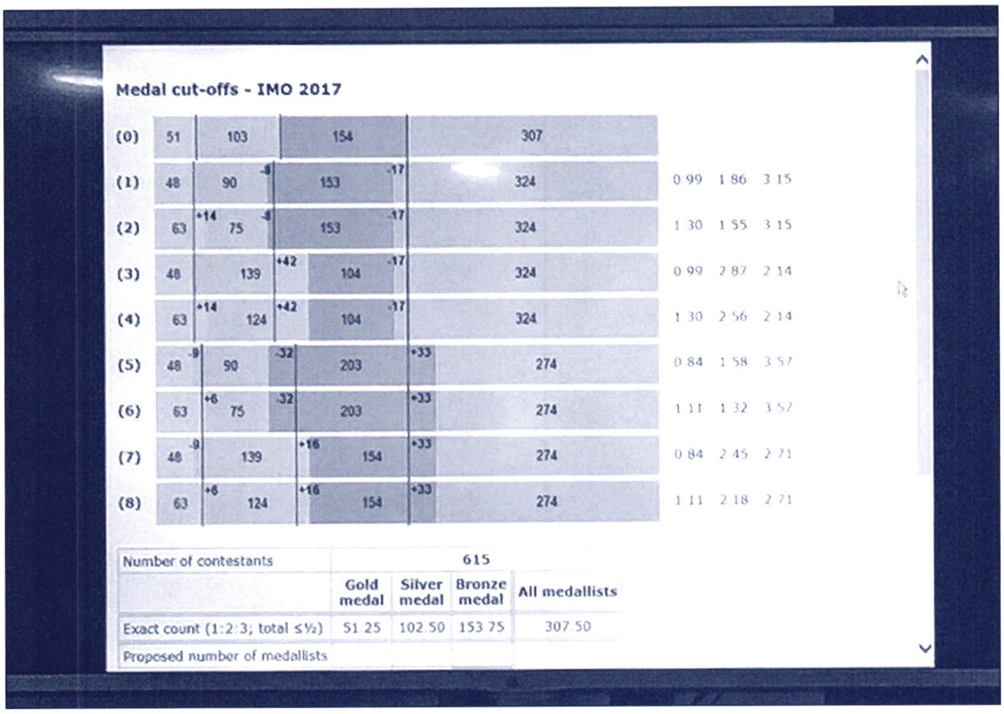
Code	P1	P2	P3	P4	P5	P6	Total
THA1		7	0	7	•	0	14
THA2		4	•	7	0	0	11
THA3		•	0	7	3	1	11
THA4		1	0	•	0	0	1
THA5		7	0	7	7	0	21
THA6		4	0	7	0	•	11
Total	**0**	**23**	**0**	**35**	**10**	**1**	**69**

• Result is known but not shown. Totals are based on revealed marks.

1 2 3 4 5 6 7 8 9 10

(Above)
Dr Wong, Mr Teh Jiun Harn and me after an coordination session
(Left)
Screen shot of the real-time scores for all countries during the coordination period

IMO implements a rigorous procedure to ensure that the marking of scripts is fair and efficient. First the coordinators appointed by the organizer and the team leaders and observers would assess the scores of the contestants separately. Then they meet at the coordination sessions to seek agreement on the final scores, one problem at a time. During the two days of coordination, the scores are updated live, but incompletely, on big screens in both venues. This creates an air full of hope and suspense during the two days, since cut-offs for medals depend on the overall distribution of the scores. If any disagreement in coordination sustains, it will be settled during the final jury meeting, in which the most important item in the agenda is voting on the cut-offs for gold, silver, bronze medals and honourable mention.



(Above) Screen shot of voting options for medal cut-offs;
(Below) Team leaders vote on cut-offs

During the final jury meeting on 21 July, a few alternatives for medal cut-offs were proposed and voted on. In the end, the option (1) was chosen by the majority and the cut-offs for medals are: 25 for Gold, 19 for Silver, and 16 for Bronze.

A Happy End

Here are the details of the performance of our contestants in this IMO:

Contestent		P1	P2	P3	P4	P5	P6	Total	Rank	Award
SGP1	Chew Xuan Da, Clarence	7	3	0	7	1	0	18	139	Bronze medal
SGP2	Ng Yu Peng	7	4	0	7	0	0	18	139	Bronze medal
SGP3	Tan Junyao, Joel	7	4	0	7	7	4	29	7	Gold medal
SGP4	Toh Shan Hong, Dylan	7	4	0	7	6	0	24	49	Silver medal
SGP5	Wang Peng Jun, Bryan	7	7	0	7	7	0	28	14	Gold medal
SGP6	Wang Jianzhi	7	4	0	2	1	0	14	342	Honourable mention
Total		**42**	**26**	**0**	**37**	**22**	**4**	**131**	**7**	**G, G, S, B, B, HM**

Though three of our students missed the cut-offs for the better medal by 1 mark each, as a team, Singapore continued to perform well, tying for 7th place (unofficially) among 110 participating countries this year.

Singapore Delegation after the 58th IMO Closing Ceremony

At the end of a 25-hour return journey, we were received by the families of the students at the Changi Airport.

In view of the medal-deciding problem 2 and 5, we may want to pay more attention to functional equations and algorithmic approach in combinatorics in our trainings in future as these topics are becoming increasingly popular in various mathematical Olympiad competitions in recent years.
(58th IMO problems are in Appendix)

Booklets of mathematical Olympiad problems from various countries

Acknowledgement:

I would like to thank the following people for their contributions and support:

- Ling Yan Hao, Teh Jiun Han, and other past contestants of Singapore IMO teams (EX-men) who helped train our contestants;
- Dr Wong Yan Loi, Dr Tay Tiong Seng, Mr Thomas Teo, Mr Zong Lixing, Mr Wang Haibin, Mr Lu Shang Yi and other SIMO committee members;
- Mr Barry Chia, Ms June Tan and other MOE CPDD Math Unit officers;
- Parents and other family members of the students,
  `and the generous sponsorship by Ministry of Education and Singapore Mathematical Society.

Ai Xinghuan
58th IMO Singapore Team Deputy Leader
14 December 2017

Appendix: 58th IMO Problems

Problem 1. For each integer $a_0 > 1$, define the sequence $a_0, a_1, a_2, \ldots$ by:

$$a_{n+1} = \begin{cases} \sqrt{a_n} & \text{if } \sqrt{a_n} \text{ is an integer,} \\ a_n + 3 & \text{otherwise,} \end{cases} \quad \text{for each } n \geqslant 0.$$

Determine all values of a_0 for which there is a number A such that $a_n = A$ for infinitely many values of n.

Problem 2. Let $\mathbb{R}$ be the set of real numbers. Determine all functions $f\colon \mathbb{R} \to \mathbb{R}$ such that, for all real numbers x and y,

$$f\left(f(x)f(y)\right) + f(x+y) = f(xy).$$

Problem 3. A hunter and an invisible rabbit play a game in the Euclidean plane. The rabbit's starting point, A_0, and the hunter's starting point, B_0, are the same. After $n-1$ rounds of the game, the rabbit is at point A_{n-1} and the hunter is at point B_{n-1}. In the n^{th} round of the game, three things occur in order.

(i) The rabbit moves invisibly to a point A_n such that the distance between A_{n-1} and A_n is exactly 1.

(ii) A tracking device reports a point P_n to the hunter. The only guarantee provided by the tracking device to the hunter is that the distance between P_n and A_n is at most 1.

(iii) The hunter moves visibly to a point B_n such that the distance between B_{n-1} and B_n is exactly 1.

Is it always possible, no matter how the rabbit moves, and no matter what points are reported by the tracking device, for the hunter to choose her moves so that after 10^9 rounds she can ensure that the distance between her and the rabbit is at most 100?

Problem 4. Let R and S be different points on a circle Ω such that RS is not a diameter. Let ℓ be the tangent line to Ω at R. Point T is such that S is the midpoint of the line segment RT. Point J is chosen on the shorter arc RS of Ω so that the circumcircle Γ of triangle JST intersects ℓ at two distinct points. Let A be the common point of Γ and ℓ that is closer to R. Line AJ meets Ω again at K. Prove that the line KT is tangent to Γ.

Problem 5. An integer $N \geqslant 2$ is given. A collection of $N(N+1)$ soccer players, no two of whom are of the same height, stand in a row. Sir Alex wants to remove $N(N-1)$ players from this row leaving a new row of $2N$ players in which the following N conditions hold:

(1) no one stands between the two tallest players,

(2) no one stands between the third and fourth tallest players,

$\vdots$

(N) no one stands between the two shortest players.

Problem 6. An ordered pair (x, y) of integers is a *primitive point* if the greatest common divisor of x and y is 1. Given a finite set S of primitive points, prove that there exist a positive integer n and integers $a_0, a_1, \ldots, a_n$ such that, for each (x, y) in S, we have:

$$a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \cdots + a_{n-1}xy^{n-1} + a_ny^n = 1.$$