
Thursday, July 16, 2009

Problem 4. Let ABC be a triangle with $AB = AC$. The angle bisectors of $\angle CAB$ and $\angle ABC$ meet the sides BC and CA at D and E , respectively. Let K be the incentre of triangle ADC . Suppose that $\angle BEK = 45^\circ$. Find all possible values of $\angle CAB$.

Problem 5. Determine all functions f from the set of positive integers to the set of positive integers such that, for all positive integers a and b , there exists a non-degenerate triangle with sides of lengths

$$a, f(b) \text{ and } f(b + f(a) - 1).$$

(A triangle is *non-degenerate* if its vertices are not collinear.)

Problem 6. Let $a_1, a_2, \dots, a_n$ be distinct positive integers and let M be a set of $n - 1$ positive integers not containing $s = a_1 + a_2 + \dots + a_n$. A grasshopper is to jump along the real axis, starting at the point 0 and making n jumps to the right with lengths $a_1, a_2, \dots, a_n$ in some order. Prove that the order can be chosen in such a way that the grasshopper never lands on any point in M .