Language: **English**

Day: **1**

*Wednesday, July 15, 2009*

**Problem 1.** Let $n$ be a positive integer and let $a_1, \ldots, a_k$ $(k \geq 2)$ be distinct integers in the set $\{1, \ldots, n\}$ such that $n$ divides $a_i(a_{i+1}-1)$ for $i = 1, \ldots, k-1$. Prove that $n$ does not divide $a_k(a_1-1)$.

**Problem 2.** Let $ABC$ be a triangle with circumcentre $O$. The points $P$ and $Q$ are interior points of the sides $CA$ and $AB$, respectively. Let $K$, $L$ and $M$ be the midpoints of the segments $BP$, $CQ$ and $PQ$, respectively, and let $\Gamma$ be the circle passing through $K$, $L$ and $M$. Suppose that the line $PQ$ is tangent to the circle $\Gamma$. Prove that $OP = OQ$.

**Problem 3.** Suppose that $s_1, s_2, s_3, \ldots$ is a strictly increasing sequence of positive integers such that the subsequences

$$s_{s_1}, s_{s_2}, s_{s_3}, \ldots \qquad \text{and} \qquad s_{s_1+1}, s_{s_2+1}, s_{s_3+1}, \ldots$$

are both arithmetic progressions. Prove that the sequence $s_1, s_2, s_3, \ldots$ is itself an arithmetic progression.



*Time: 4 hours and 30 minutes*
*Each problem is worth 7 points*