IMO 2007

Ha Noi, Vietnam

Day 1 - *25 July 2007*

1 Real numbers $a_1, a_2, \ldots, a_n$ are given. For each i, $(1 \leq i \leq n)$, define

$$d_i = \max\{a_j \mid 1 \leq j \leq i\} - \min\{a_j \mid i \leq j \leq n\}$$

and let $d = \max\{d_i \mid 1 \leq i \leq n\}$.

(a) Prove that, for any real numbers $x_1 \leq x_2 \leq \cdots \leq x_n$,

$$\max\{|x_i - a_i| \mid 1 \leq i \leq n\} \geq \frac{d}{2}. \qquad (*)$$

(b) Show that there are real numbers $x_1 \leq x_2 \leq \cdots \leq x_n$ such that the equality holds in (*).

2 Consider five points A, B, C, D and E such that $ABCD$ is a parallelogram and $BCED$ is a cyclic quadrilateral. Let ℓ be a line passing through A. Suppose that ℓ intersects the interior of the segment DC at F and intersects line BC at G. Suppose also that $EF = EG = EC$. Prove that ℓ is the bisector of angle DAB.

3 In a mathematical competition some competitors are friends. Friendship is always mutual. Call a group of competitors a *clique* if each two of them are friends. (In particular, any group of fewer than two competitiors is a clique.) The number of members of a clique is called its *size*.

Given that, in this competition, the largest size of a clique is even, prove that the competitors can be arranged into two rooms such that the largest size of a clique contained in one room is the same as the largest size of a clique contained in the other room.

