47th **I**NTERNATIONAL
MATHEMATICAL
OLYMPIAD
SLOVENIA 2006

day: **2**

language: **English**

13 July 2006

Problem 4. Determine all pairs (x, y) of integers such that

$$1 + 2^x + 2^{2x+1} = y^2.$$

Problem 5. Let $P(x)$ be a polynomial of degree $n > 1$ with integer coefficients and let k be a positive integer. Consider the polynomial $Q(x) = P(P(\ldots P(P(x)) \ldots))$, where P occurs k times. Prove that there are at most n integers t such that $Q(t) = t$.

Problem 6. Assign to each side b of a convex polygon P the maximum area of a triangle that has b as a side and is contained in P. Show that the sum of the areas assigned to the sides of P is at least twice the area of P.

Time allowed: 4 hours 30 minutes
Each problem is worth 7 points