46^{th} International Mathematical Olympiad

Second Day

Merida, Mexico, Thursday 14 July 2005

Language: English

Problem 4. Consider the sequence $a_1, a_2, \ldots$ defined by

$$a_n = 2^n + 3^n + 6^n - 1 \quad (n = 1, 2, \ldots).$$

Determine all positive integers that are relatively prime to every term of the sequence.

Problem 5. Let $ABCD$ be a given convex quadrilateral with sides BC and AD equal in length and not parallel. Let E and F be interior points of the sides BC and AD respectively such that $BE = DF$. The lines AC and BD meet at P, the lines BD and EF meet at Q, the lines EF and AC meet at R. Consider all the triangles PQR as E and F vary. Show that the circumcircles of these triangles have a common point other than P.

Problem 6. In a mathematical competition 6 problems were posed to the contestants. Each pair of problems was solved by more than $\frac{2}{5}$ of the contestants. Nobody solved all 6 problems. Show that there were at least 2 contestants who each solved exactly 5 problems.

Time allowed: 4 hours 30 minutes

Each problem is worth 7 points