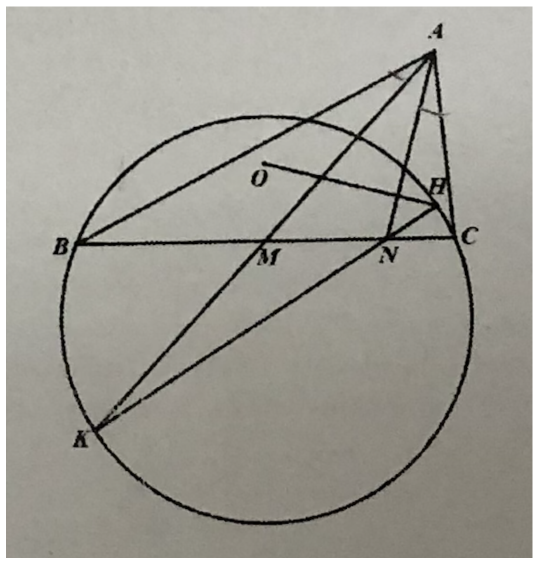
Questions

Day 1

1. Find all positive integer n such that the number $3^n + n^2 + 2019$ is a perfect square.

2. Let O and H be the circumcenter and the orthocenter of an acute triangle ABC with $AB \neq AC$, respectively. Let M be the midpoint of BC, and let K be the intersection of the line AM and the circumcircle of ΔBHC, such that M lies between A and K. Let N be the intersection of the lines HK and BC. Show that if $\angle BAM = \angle CAN$, then $AN \perp OH$.



3. Let $S = \{(i,j)|i,j = 1,2,...,100\}$ be a set consisting of points on the coordinate plane. Each element of S is coloured one of four given colours. A subset T of S is called colourful if T consists of 4 points with distinct colours, which are the vertices of a rectangle whose sides are parallel to the coordinate axes. Find the maximal number of colourful subsets that S can have, among all legitimate colouring patterns.

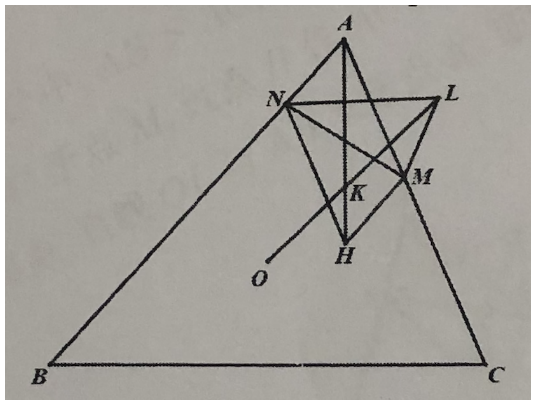
4. Let n be a given integer such that $n \geq 2$. Find the smallest real number λ with the following property: for any real numbers $x_1, x_2, ..., x_n \in [0,1]$, there exist integers $\epsilon_1, \epsilon_2, ..., \epsilon_n \in \{0,1\}$ such that the inequality

$$\left| \sum_{k=i}^{j} (\epsilon_k - x_k) \right| \leq \lambda$$

holds for all pairs of integers (i,j) where $1 \leq i \leq j \leq n$.

Day 2

1. Let ABC be an acute triangle such that $AB > AC$, with circumcenter O and orthocenter H. Let M and N be points on AC and AB, respectively, such that $HN//AC$ and $HM//AB$. Let L be the reflection of H in MN, and the lines OL and AH intersect at K. Show that the points K, M, L, N are concyclic.



2. Let n be a given integer such that $n \geq 2$. For any n positive real numbers $a_1, a_2, ..., a_n$ such that $a_1 \leq a_2 \leq \cdots \leq a_n$, show that

$$\sum_{1\leq i<j\leq n} (a_i + a_j)^2 \left(\frac{1}{i^2} + \frac{1}{j^2}\right) \geq 4(n-1) \sum_{i=1}^{n} \frac{a_i^2}{i^2}.$$

3. Show that for any positive integer k, there are at most finitely many sets T with the following properties:

   - T consists of finitely many prime nunbers;
   - $\prod_{p\in T} p | \prod_{p\in T}(p+k)$.

4. A set S is called a good set if $S = \{x, 2x, 3x\}$ for some real number x. For a given integer $n \geq 3$, find the maximal number of good subsets that an n-element set of positive integers may have.