MATHEMATICS ENRICHMENT PROGRAMME

SINGAPORE MATHEMATICAL SOCIETY

1. An Introduction to Cryptology

1.1. **Instructor.** Dr. KU Cheng Yeaw is a Senior Lecturer at the Department of Mathematics, Nanyang Technological University (NTU). He received his Ph.D. from Queen Mary, University of London in 2005. Before joining National University of Singapore (NUS), he was a Harry Bateman Research Instructor at the Department of Mathematics, California Institute of Technology (2005–2008). His main research area is in Combinatorics and Graph Theory. Some of his latest research projects include the extension of the Gallai-Edmonds Structure theorem to general roots of the matching polynomials and solving the Ku-Wales conjecture of the eigenvalues of the Derangement Graph. Dr. Ku has also been awarded the Faculty Teaching Excellence Award for the academic years 2010–2014 in NUS. He is currently the Assistant Chair of School of Physical and Mathematical Sciences, and Deputy Head of Division of Mathematical Sciences of NTU.

1.2. **Course Description.** This module introduces the students to the art and science of secret codes. Beginning with the mathematical background (modular arithmetic), the students are taught how to encrypt and decrypt messages using the shift, transportation, and affine ciphers.

The module concludes with a Cryptology Tour of Singapore in which the students must decrypt the secret messages which will lead them to a tourist attraction in Singapore.

2. The Königsberg Bridge Problem and the Chinese Postman Problem

2.1. **Instructor.** Professor KOH Khee Meng obtained his Ph.D. from the University of Manitoba in Canada in 1971. He is a Professor in the Department of Mathematics at the National University of Singapore. Among several other significant appointments, Prof Koh was the chairman of the Singapore International Mathematical Olympiad Committee (1991–93), a council member of the Institute of Combinatorics and Its Applications (International) (1995–97) and the president of the Singapore Mathematical Society (1996–98). He has also won over 20 Teaching Awards from the Faculty of Science, and more recently the prestigious Outstanding Educator's Award in NUS. Prof Koh specializes in Combinatorics and Graph Theory and has had many papers published in international

scientific journals. He is coauthor of the books: *Principles and Techniques in Combinatorics*, *College Mathematics Volumes I & II*, *Counting*, and *Chromatic Polynomials and Chromaticity of Graphs*.

2.2. **Course Description.** Theory of Graphs dates back to 1736 when the great Swiss Mathematician Leonhard Euler devised an innovative approach to solve the problem involving seven bridges in Königsberg. Graph theory has come a long way since then. In this workshop, the instructor will give a brief introduction to the graph theory with the Königsberg Bridge Problem and the Chinese Postman Problem.

3. Probability Through Games

3.1. **Instructor.** Dr. David CHEW is a Senior Lecture of the Department of Statistics and Applied Probability, National University of Singapore (NUS). He received his Ph.D. from NUS in 2006. While pursuing his Ph.D. with the Department of Mathematics, he was awarded the Faculty's Outstanding Teaching Assistant Award thrice and the Graduate Research Award in 2005. Upon completion of his Ph.D. programme, he underwent postdoctoral training at the University of Southern California under a NUS-Overseas Postdoctoral Fellowship. He joined the Department of Statistics and Applied Probability as a lecturer in Mar 2009, and he is now a senior lecturer in the department. His research interest lies in Computational Biology, particularly in the area of biological sequence analysis.

3.2. **Course Description.** A gambler's dispute in 1654 led to the creation of a mathematical theory of probability by two famous French mathematicians, Blaise Pascal and Pierre de Fermat. Today, probability is a thriving mathematical area and has many potential applications in real problems. In this talk, we give an introduction to important concepts and selected famous problems in probability. Through simple and fun activities, it is hoped that participants will be able to gain insights and appreciation of these problems and concepts.

4. Combinatorial Games

4.1. **Instructor.** Associate Professor Victor TAN received his B.Sc. (Hons) degree from the National University of Singapore in 1987 and his Ph.D. degrees from the University of California, Los Angeles, in 1996. He has been teaching in the Department of Mathematics, NUS since then. He is a multiple winner of the Teaching Excellence Awards at the Faculty and University level, including the most prestigious Outstanding Educator Award in 2007. He is a fellow of the Teaching Academy in NUS. Prof Tan's research interests are on Algebra and Number Theory. He has also keen interest in using IT in Mathematical

teaching. In terms of outreach programme, Prof Tan has also conducted mathematical enrichment programmes in many schools. He is currently the President of the Singapore Mathematical Society and the Deputy Head of the Department of Mathematics, NUS.

4.2. **Instructor.** Dr. NG Kah Loon is currently a Senior Lecturer at the Department of Mathematics, NUS. He also holds the appointment of Assistant Dean (Undergraduate Program) in the Faculty of Science's Deans Office. Dr Ng received his Bachelor, Master and Ph.D. degrees from NUS and has two years of Postdoctoral experience in USA. Dr Ng's research interest is in the area of Graph Theory, in particular the applications of Graph Theory to the areas of Social and Biological Sciences. Some of the courses that he has taught in NUS includes Linear Algebra, Combinatorics and Graph Theory.

4.3. **Combinatorial Games.** This is one of our most successful modules in terms of student response. It consists of a series of hands-on games and puzzles. While the students have fun with these games, they learn how certain ideas from combinatorial mathematics — graph theory, tiling, and symmetry — provide the underlying principles of their solutions.

5. Fractals

5.1. **Instructor.** Associate Professor Victor TAN received his B.Sc. (Hons) degree from the National University of Singapore in 1987 and his Ph.D. degrees from the University of California, Los Angeles, in 1996. He has been teaching in the Department of Mathematics, NUS since then. He is a multiple winner of the Teaching Excellence Awards at the Faculty and University level, including the most prestigious Outstanding Educator Award in 2007. He is also a fellow of the Teaching Academy in NUS. Prof Tan's research interests are on Algebra and Number Theory. He has also keen interest in using IT in Mathematical teaching. In terms of outreach programme, Prof Tan has also conducted mathematical enrichment programmes in many schools. He is currently the President of the Singapore Mathematical Society and the Deputy Head of the Department of Mathematics, NUS.

5.2. **Course Description.** This module introduces students to some of the exciting mathematics behind some of the most stunningly beautiful images in mathematics — fractals. The concept of complication (and beauty) arising from infinite iteration of simple operations will be reinforced through the Iterated Function System (IFS). This is an important result of Michael Barnsley and is useful in modeling nature. Students will be able to appreciate its power by using the IFS to model trees, leaves, mountains and etc.

6. What Are the Odds?

6.1. **Instructor.** Associate Professor YAP Von Bing received his B.Sc. (Hons) degree in Mathematics from NUS in 1996, an M.Sc. in Applied Mathematics from NUS in 1997, and a Ph.D. in Statistics from University of California in 2002. He is interested in the application of statistics to scientific problems, especially those arising in evolutionary biology and bioinformatics, and also the communication of mathematical and statistical ideas in the context of science education to young people. He has been awarded the Faculty and University Teaching Excellence Award. He is currently a member of the Teaching Academy and the Assistant Dean of Science at NUS.

6.2. **What are the Odds?** The term "odds" is used widely to describe and predict uncertain events. We will look quite carefully at this idea through the more familiar notions of population proportions and probabilities. A variety of examples from epidemiology and sports will be used to illustrate the ideas.

7. Introduction to Projective Geometry

7.1. **Instructor.** Dr. WANG Fei is currently a Senior Lecturer at the Department of Mathematics, NUS. He received his B.Sc (Hons) from the National University of Singapore in 2004. He joined the Department of Mathematics of NUS since 2006 as a teaching assistant while pursuing his Ph.D. With the passion in teaching, he won the Excellent Teaching Assistant Award from the Faculty every year. After he received his Ph.D in 2011, he is promoted to be a lecturer in the Department of Mathematics. Dr. Wang has also been awarded the Faculty Teaching Excellence Award for the academic years 2015–2017, and University Annual Excellence Teaching Award in 2017. He is now also an Executive Council member of Singapore Mathematical Society, and in charge of mathematics enrichment programme and Singapore Mathematics Project Festival. His research area is algebraic geometry.

7.2. **Introduction to Projective Geometry.** In projective geometry, we are no longer interested with the distance and area. Instead, we focus on the configuration of points and lines. By starting from the "points at infinity", we will study the projective transformations on the plane, cross-ratio theorem, and the Riemann sphere.

8. Mathematics and Origami

8.1. **Instructor.** Dr. GWEE Hwee Ngee is the Head of Mathematics Department at the School of the Arts (SOTA). Prior to her current appointment, she was the Principal Consultant for Mathematics at Hwa Chong Institution (High School Section). She obtained

her Doctorate of Education and Masters of Education from University of Western Australia. She is passionate in Math and Origami and has mentored various Math projects involving Origami, one of which obtained the Foo Kean Pew Memorial Prize (Senior Section) in 2008. She has also conducted several workshops involving Mathematics and Origami at HCI and MEW. Dr. Gwee enjoys making connections across disciplines and sees Mathematics as an Art. She would like to inspire the way students use and learn Mathematics and hope that they, too, would appreciate Mathematics as an Art.

8.2. **Course Description.** This Origami workshop will cover geometry concepts, such as symmetry, polygons, polyhedras and Euler's formula. Students will be able to make their own 3D modular Origami models during the course.

9. Introduction to Partitions

9.1. **Instructor.** Dr. TOH Pee Choon received his Ph.D. from the National University of Singapore (NUS) in 2007. He is currently an Associate Professor at the National Institute of Education (NIE), Nanyang Technological University. Prior to joining NIE, he was a lecturer at NUS. Dr Toh has received Teaching Awards from both NUS and NIE. His main research interest is Number Theory, specifically the theory of partitions, modular forms and elliptic functions. Dr Toh has keen interest in mathematics education and has worked with various schools and research projects in the area of problem solving, justifications and proofs, and the teaching of mathematics at the undergraduate level.

9.2. **Course Description.** A partition of an integer n is a way of writing n as a non-increasing sum of positive integers. The basic problem is to count the number of different ways this can be done for each integer n. For example, the integer 3 can be written as 3 or $2+1$ or $1+1+1$, giving us three partitions of 3. Mathematicians define $p(n)$ as the number of partitions of n, and so we have $p(3) = 3$. Similarly, $p(4) = 5$ because 4 can be written as $4, 3+1, 2+2, 2+1+1$ and $1+1+1+1$. However the function $p(n)$ grows rapidly as n increases. We have $p(50) = 204\,226$. The aim of this workshop is to introduce participants to various arithmetic and combinatorial properties of $p(n)$.