What Should You Do If You Fall Into a BLACK HOLE?

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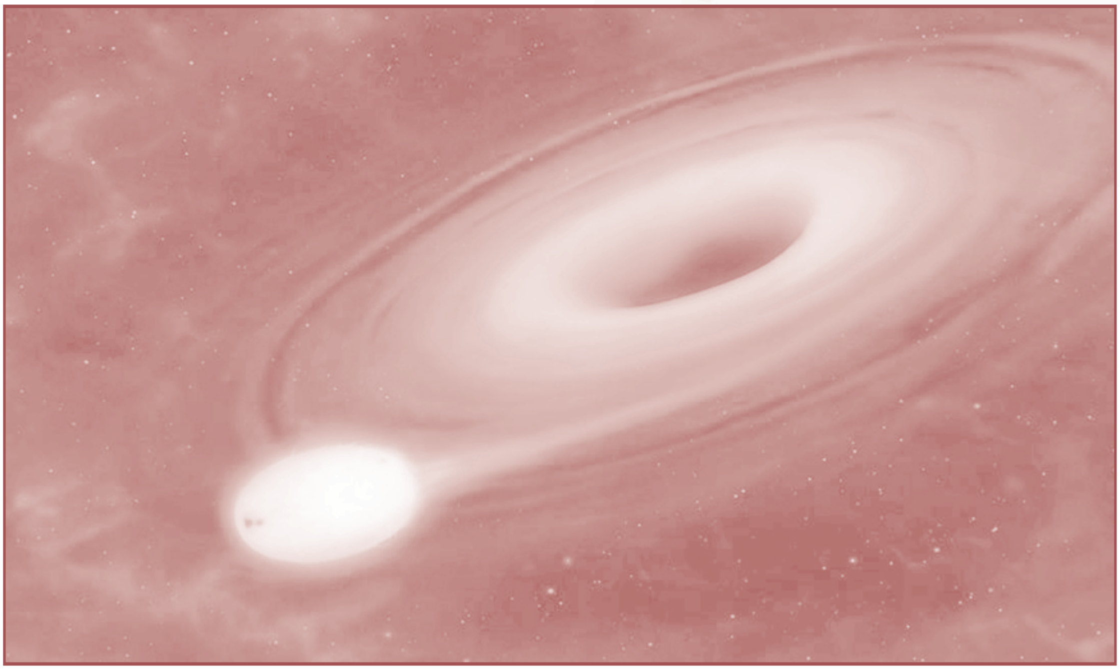

ABSTRACT

What should you do if you are unfortunate enough to fall into a black hole? The answer to this is still controversial, which means that it is still a matter for research. Here I will only discuss the simplest answer, derived using only the theory of General Relativity. The answer itself will probably surprise you. But what is just as interesting is that *it is possible to give any answer at all*! After all, we cannot do any experiments with black holes and we cannot see inside them. There is only one way to do it: using mathematics, of course.

What is Spacetime?

Albert Einstein [1879-1955] made many very great contributions to science[1]. But probably the greatest, and the one that is most important for theoretical physics research today, is this: *he explained that the geometry of spacetime is not as simple as it might be: spacetime is bent!* But what is spacetime?

Spacetime is just the set of all possible events, or happenings. For example, you were born at [approximately] one specific time in [approximately] one specific place. That is an event. Events are obviously specified by three spatial coordinates, say (x, y, z), plus one time coordinate, usually called t. So spacetime is a four-dimensional set. It is traditional to draw t to be on the vertical axis, and one of the spatial coordinates, say x, on the horizontal axis. Actually, it is traditional also to use not t, but rather ct, where c is the speed of light in a vacuum. We do this so that the units on both axes are the same, namely units of distance, say metres.

Any given object, like you or your friend sitting next to you, has a history. This history is represented in spacetime by a line, which need not be straight, but which is mostly vertical in spacetime. These lines are called *worldlines* [See Figure 1]. Worldlines are mostly vertical for several reasons [see below]. One of them is to stop anyone from travelling backwards in time, with paradoxical consequences familiar to anyone who has seen the relevant movies [such as the ancient "Back to the Future" series].

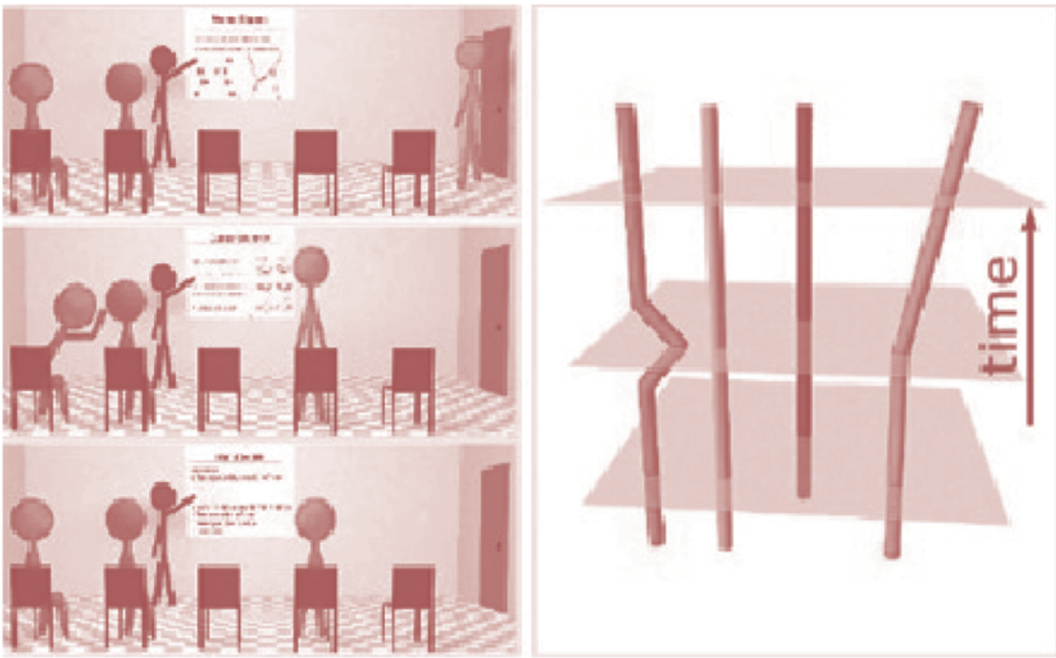

Figure 1: Spacetime diagram
(Source: An introduction to String Theory, by Steuard Jensen)

[1]See his entry in wikipedia; I hope you will do that for every topic in this article....

Some examples of worldlines:

• A perfectly stationary object has a perfectly vertical, *straight* worldline.

• An object moving at constant speed has a *straight* worldline which is however inclined at some angle to the axes.

• An object which is accelerating has a *curved* worldline which bends more and more away from the vertical [time] axis.

Spacetime Geometry

Ordinary three-dimensional geometry is familiar to you, and it has a number of rules. One famous rule in ordinary geometry is

The shortest curve between two points in three-dimensional space [ie, NOT in spaceTIME] is the straight line joining them.

Spacetime also has a geometry, but it is very different from ordinary geometry! For example, we have the following rule, which is an explicit realisation of the "mostly vertical" rule for worldlines:

Rule # 1. No worldline is ever allowed to make an angle of more than 45 degrees with the vertical axis [that is, the ct axis].

If you think about it, you will realise that this rule prohibits anything from moving faster than light [hint: a ray of light moving along the x axis has a worldline described by $x = ct$.] And this is a good rule, because indeed nobody has ever seen anything moving faster than light! But there is nothing analogous to this in ordinary geometry.

Now just as we have a concept of distance in ordinary geometry, so also there is a concept of "distance" in spacetime. But distance in spacetime has to incorporate both ordinary distance *and* separation in time. For example, the Pharaoh Sneferu was born both long ago [over 4600 years ago] and far away [from Singapore], and "distance" in spacetime has to allow for both possibilities. So how "distant" was the birth of Sneferu from us in Singapore, right now? The definition of distance in spacetime, in the simplest possible case, is given by

$$d = \sqrt{|c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2|},$$

where Δx is the difference of the x-coordinates of two events, and so on. This looks like Pythagoras' theorem. But notice all of those strange minus signs [and absolute value signs, which ensure that we never take the square root of a negative number here]. They make a tremendous difference! Still, we see that if $\Delta t = 0$ [which means that we are talking about two events happening at the same time, though possibly in different places], then this formula does give you back exactly the ordinary three-dimensional Pythagoras theorem, so the formula does make sense. [It also gives a sensible answer for two events happening at different times but at the same place.]

Now given any worldline, like the one describing the history of your life, we can work out its length using this formula. But what is the meaning of this "length"? The answer is given by

Rule # 2. The distance along a worldline measures the time actually experienced by a person or object with a history described by that worldline.

This simple rule has consequences which seem shocking at first. To give a famous example: suppose two identical twin sisters are born in Arles [France] on 21 February 1865. One of them stays in Arles and never moves out. The other gets into a spaceship and travels around the solar system at extremely high speeds. In early August 1997 she returns home to visit her sister[2].

Jeanne Calment (1865-1997)
(Source: http://www.grg.org/JCalmentGallery.htm)

The two worldlines are very very different: the first sister's worldline is almost perfectly straight and vertical, whereas the second sister's worldline is bent around, recording her visits to other planets. *Obviously the two worldlines cannot have the same length*, though they have the same endpoints. So, according to Rule # 2, the two sisters, even though they are identical twins, *have different ages* when they meet in 1997. The effect is normally very small, but it has in fact been observed in many many experiments[3].

[2] Jeanne Calment, who had the longest confirmed life span of any human being in history. The real Jeanne did not have an identical sister, unfortunately.

[3] In fact, the GPS system used by the smartphone which is probably in your pocket is programmed to allow for this effect; the GPS satellites have very different worldlines from that of your phone.

Many readers are probably familiar with this fact. You may however be surprised to learn the mathematics underlying this situation. Let's try to answer the question: which of the two sisters is the younger? This can easily be deduced from the next Rule of spacetime geometry[4]:

Rule # 3. The LONGEST of all worldlines connecting two given events is the straight worldline joining them.

This is of course the *exact opposite* of the usual rule in three-dimensional space! [It happens because of all those minus signs in the formula above.] Now if you think about it, or draw a picture, you will see that the sister who stayed home had a worldline which was almost straight. So her worldline is very much *longer* than that of the sister who went travelling. According to Rule # 2, that means that she, the stay-at-home sister, is old; the traveller returns home, still a beautiful young girl, only to find that her "identical" sister is 122 years old! This is a lot less shocking when viewed geometrically: of course differently shaped lines have different lengths.

What is a Black Hole?

Now if you look at the formula above, you will see that the geometry is the same everywhere and at all times — nothing in the formula depends on position or time. That's just like Pythagoras' theorem for plane geometry, which is indeed the same everywhere. But in mathematics we often consider spaces in which the geometry does vary from place to place. Consider for example the back of your hand: it has various bumps and valleys on it. In *Differential Geometry* we say that the "metric", the formula used to compute distances, is a function of position on your hand.

Einstein realised that the same thing can happen in spacetime: the coefficients in the formula for spacetime distance can become functions of x, y, z, t. This marked the transition from *Special* to *General* relativity[5]. The details are [very] complicated, but the crucial thing to understand is this: *Einstein kept Rules # 1, 2, 3 intact*. That's all you really need to know to understand what a black hole is, and what you should do when you fall into one.

Now, unlike constants, functions don't always behave nicely. Consider for example the function $y = 1/x^2$. We all know that it is not defined at $x = 0$: there is a sharp spike there. In General Relativity, it turns out that, when certain kinds of stars explode in a supernova, the functions in the spacetime distance formula behave like that: the spacetime geometry inside the supernova develops a "spike". Such a spike is called a *singularity*. In Figure 2, the singularity is marked as the jagged line ["$r = 0$"]. Just as $y = 1/x^2$ cannot be defined at $x = 0$, so also space-

[4]This Rule is actually a theorem: it follows from the definition of spacetime distance.

[5]Amazingly, the GPS system used by your smartphone is *also* programmed to allow for the fact that the GPS satellites move through a region of spacetime with a geometry which differs slightly from the geometry here on the surface of the Earth!

time is not well-defined at a singularity: in fact, spacetime simply ceases to exist at the singularity.

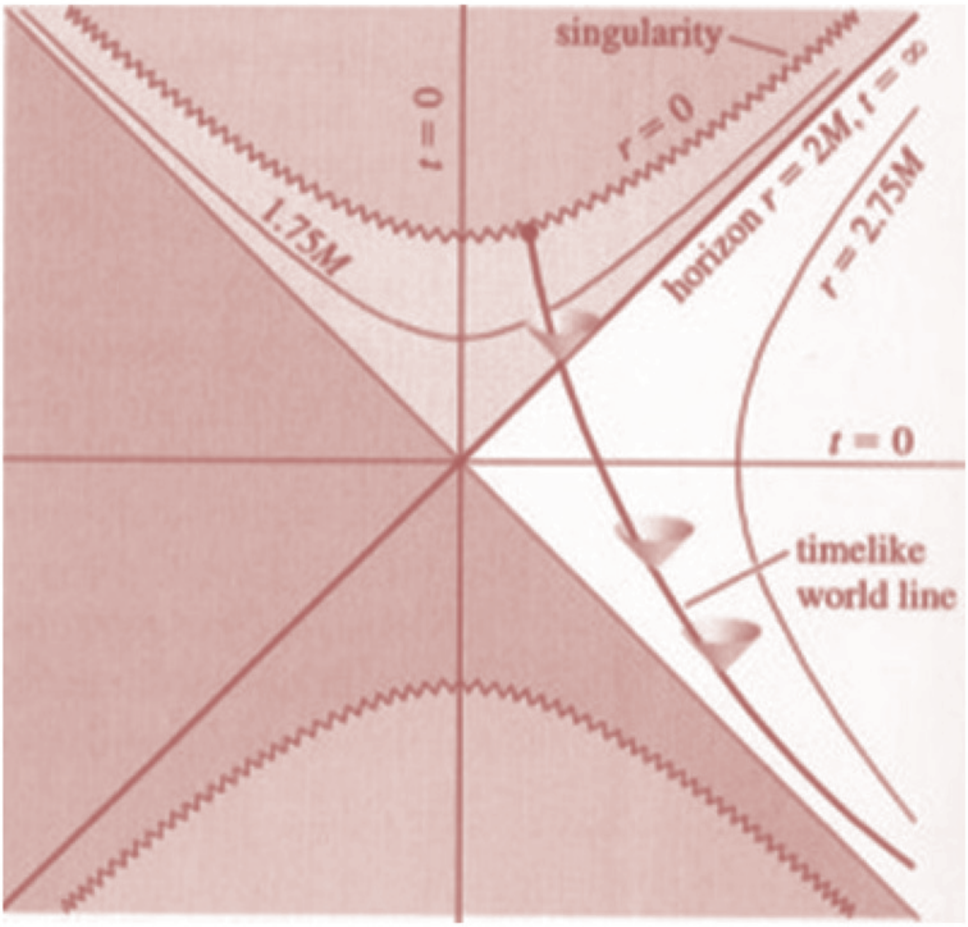

Figure 2: Spacetime geometry
(Source: An introduction to Einstein's general relativity, by James B. Hartle)

Now notice something very surprising and alarming in Figure 2: the singularity runs roughly *horizontally*, not vertically. That means that if you are near to it [*near in time, not space*], you cannot escape it. Can you escape arriving at next Monday? No, because Monday is a time, not a place. Likewise *the singularity is like a time, not a place!*

Next, notice that diagonal line [at 45 degrees] in Figure 2 marked as the *event horizon*. Imagine that you are above [that is, later than] that line. Now try to avoid the singularity; that is, try to draw a worldline that avoids hitting the singularity. The only way to do it is to draw a line that makes an angle exceeding 45 degrees with the vertical. But this is forbidden by Rule # 1! *Once you are inside the event horizon, you cannot go outside again.*

Any spacetime geometry that has this strange combination of a singularity with an event horizon is called, by definition, a black hole.

How much time do you have before you cease to exist? Well, that is something which can be computed in detail, and we will return to it below. But it's clear from Figure 2 that all worldlines inside the event horizon terminate on the singularity;

so, in view of Rule # 2, the time you have left alive is certainly *finite*. [Outside the event horizon, it *is* possible to avoid the singularity without violating Rule # 1: look carefully.]

At this point, I would like to draw your attention to something. Most people have heard about black holes, and usually the story runs like this.

"Black holes are places where gravitational attraction is so strong that nothing, not even light, can escape from their clutches. Once you get inside, you are pulled inexorably to the centre and destroyed there.".

Well, just about everything in these sentences is either very misleading or wrong. You see that in my description above, I never mentioned "gravitational attraction" at all! The reason you cannot escape is not because you are being "pulled" anywhere: that would be as ridiculous as saying that you are being "pulled" into next Monday! Furthermore, I did not talk about singularities at the "centre" of the black hole, because there is no singularity there: the singularity is a time, not a place; it is certainly not at the "centre" [and indeed, from the diagram, it is not clear that the black hole *has* a "centre".] You should not think of the black hole as a nasty thing that is trying to keep you a prisoner. *You should think of it as a place where the shape of [spacetime] geometry is very unusual*, so unusual that it does not allow you to live indefinitely!

Black holes are an exercise in geometry.

So What Should You Do?

In the preceding section I urged you not to think of the black hole as a place where "gravitational attraction" is trying to "pull you in". You might have thought that I was being rather pedantic in not allowing myself to talk that way. But in fact, this intuition about "being pulled in" will lead you to do exactly the wrong thing if you ever fall into a black hole! Let's reason it out.

We agreed that, having fallen in, you are not going to get out, and in fact your remaining lifetime is finite. But you can still try to live as long as you possibly can. In fact, for a really huge black hole — and some real black holes are indeed huge, up to 20 billion times as massive as the Sun — you might be able to live for days after falling in, if you behave in the right way. This is a familiar situation: here on earth, some people [not enough] eat sensibly and exercise a lot in order to extend their lives.

So suppose that you remain calm and try to maximise your remaining lifetime. According to Rule # 2, that means that you should try to maximise the length of your worldline. According to Rule #3, that means that *you should keep your worldline as straight as possible*. But [see the discussion at the end of the first section, above] that means that you should accelerate as little as possible.

In other words, you should just accept your fate and avoid all unnecessary exertion. If you panic and try to fight your way out, your worldline will bend and, according to Rule # 3, that will shorten it, and, with it, your remaining lifetime!

In summary:

QUESTION: What should I do if I fall into a black hole?

ANSWER: Nothing. Go with the flow.

Now you see what I meant earlier. If you really believed all that nonsense about being "pulled into the singularity", then the natural thing to do would be to try to fight your way out. And that would be exactly the wrong thing to do!

In Conclusion....

I should say that everything I have said here is based on what we call "Classical" General Relativity[6]. That means that I have not taken into account the effects of quantum mechanics. Actually, at the moment there is a fierce controversy raging among experts as to what actually does happen if you fall into a black hole, taking quantum effects into account: this is called the "Firewall Controversy" [2]. In my opinion — but, as with any controversy, I could be completely wrong — the outcome of this debate will not materially change what I have written above.

But the main thing I want Medley readers to note is this. Black holes are essentially geometric objects; which means that they are essentially *mathematical* objects. This is really just an example of something which is obvious when you think about it, yet rarely given sufficient emphasis: *everything* we know about the Universe outside our planet and its immediate neighbourhood is known through mathematics. We do not see inside black holes, but then we do not see inside planets and stars either. All we *see* are tiny points of light through telescopes [and not even that in the case of more remote objects, which are detected through X-rays and other kinds of signals]. The only thing we can do is to *deduce* what is happening inside planets, stars, and black holes. And that process of deduction, which has been spectacularly successful, *could not even begin without mathematics!*

References

[1] GFR Ellis and Ruth Williams, Flat and Curved Space Times, Oxford University Press, 1988, revised 2000.

[2] http://www.nature.com/news/astrophysics-fire-in-the-hole-1.12726

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[6] For further reading on this subject, I recommend [1].