A Conversation with
SMS Distinguished Visitor: Imre Leader

Yap Von Bing

Our 10th Distinguished Visitor Imre Leader is a Professor of Pure Mathematics at the University of Cambridge. His research work has concentrated on Graph Theory and Combinatorics particularly in isoperimetric inequalities, extremal combinatorics and Ramsey Theory. Professor Imre is not only a world-renowned mathematician but is deeply passionate about Mathematics Olympiads and was the Chief Coordinator and Problems Group Chairman for the International Mathematical Olympiad in 2002.

On behalf of Mathematical Medley, Yap Von Bing (VB) interviewed Professor Leader (IL) on 28 August 2013 at the NUS High School of Mathematics and Sciences. The following is an edited version of the interview.

Yap Von Bing (VB): Thank you very much for agreeing to be interviewed by the SMS, Professor Leader.

Professor Leader (IL): Call me Imre.

VB: How can the mathematics community popularise the subject among school children in Singapore?

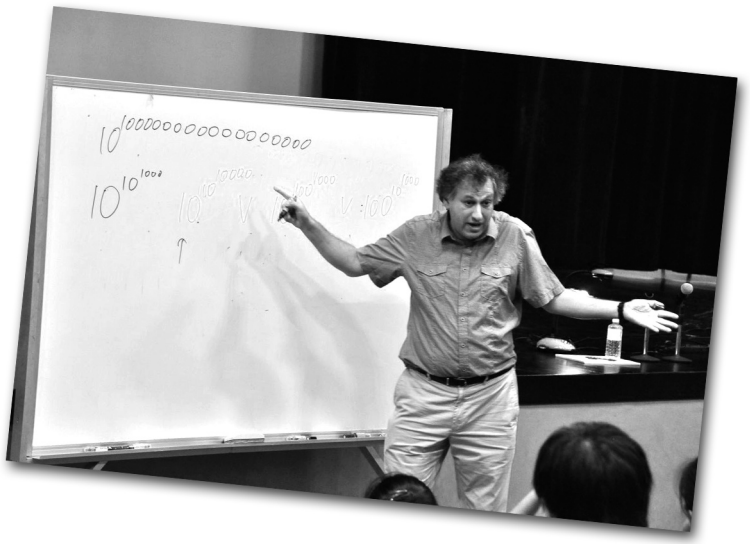
IL: I think there are two aims of popularising maths. One is that people will learn to do routine maths, or maths they need for other subjects like engineering. This is probably quite automatic...most students will realise that to do physics or economics, they need some maths. That is fine. The question is how to get students passionate about mathematics. And here I think giving them interesting thinking questions is great. And here I think Singapore is doing a fantastic job. The idea that you have primary Olympiad, that you ask primary school kids questions which are not just "apply this boring algorithm, but think of it" is just fantastic. I think certainly UK and American have a lot to learn from Singapore.

VB: The SMS holds masterclasses for 13 or 14 year-old students on interesting topics. The style is roughly like what you mentioned, asking questions and solving problems. What are some other methods of engagement?

IL: Masterclasses are a great idea. Perhaps one important thing is to get teachers on board. It's always fine to have a few people to get masterclasses from a university and so on. But to really have an impact, it's surely important to get lots of high school teachers interested in asking non-routine things. So any kind of outreach programme which gets teachers involved has to be the key. The more you can get teachers enthusiastic in asking non-routine things the better. Masterclasses may end up enthusing 0.1% of students. If you want to enthuse 20%, it has to be done in schools.

VB: What kind of mathematics should undergraduates learn in the university, especially those who do not intend to become professional mathematicians?

IL: Most students who don't want to become mathematicians are going to have two kinds of needs. First there are students who will go on to be using maths a lot. So this would mean people going to engineering, physics, chemistry, economics. In this case, it's fairly clear, just tell them what they need and so on. Beyond that, if they are not going to use maths a lot, they are going to say banking or something like that, it is most important that they learn to think. If they want to be a computer programmer, then much more important than any mathematical or programming facts they've learnt, is the ability to think. That's why normally software companies like maths graduates, because they believe, rightly or wrongly, that the most important thing is that the students come out being able to think clearly, logically, analytically.



Professor Leader delivering his public lecture on "Think of a Number" under the SMS Distinguished Visitor Program

VB: What is a better way to encourage these thinking skills in a mathematics course? I know it sounds like a weird question, because when you do higher mathematics, it's almost taken for grant that you are training thinking skills. But in practice, there will be room for improvement.

IL: Unfortunately, very often, in many places around the world, even in universities, people teach things in a rote and routine way. So very often there is a culture where students sit in lectures and learn some theorems, and then on the assignment or homework sheet, to answer the questions, what you do is, the student reads the question, looks up a theorem in the lecture notes, inserts numbers and gets the solution. That's a real shame. It's important to give students some thinking questions. And perhaps the most important thing, when you give students thinking questions, you tell the students it's OK to not succeed. It's good for students to think about the questions. If they solve the questions, it's a nice bonus. Very often, maths departments are reluctant to give students thinking questions because "Oh, the students will taste failure, they will feel bad or discouraged." But actually that encourages rote thinking and probably for going to the real world, they should be brave and willing to take risk, be willing to try some problems, be willing to try some approaches where you can't see initially that it led to the solution.

VB: Is that a problem even in Cambridge?

IL: Yes, absolutely. There are lots of courses that have great sheets of problems, but lots have pretty bad sheets of problems, where really it's just all routine. Even sometimes the sheet of problems has at the end some extension problems, but they are just bigger calculations. That's a real shame. It would be nice if every exercise sheet has some more interesting questions that make students think. It's a question of timing. Sometimes, a lecturer for a course is in a rush and just copies the sheets from previous lecturers. That way, there isn't time to develop new questions.

VB: It's very challenging to make thinking questions.

IL: That's a good point. It is very easy for me to sit here and say "Do more thinking questions." It's very hard --- anyone knows who has tried --- it's very hard to invent thinking questions. To put it another way, it's easy to invent routine questions; it's easy to invent thinking questions that are way too hard unless you have a superstar student. Thinking questions take lots of work, lots of time to do, definitely.

VB: If you want to encourage our colleagues (in Mathematics or Statistics) to do more thinking questions in exercises and examinations, would you think one way is to find textbooks that go in this direction? Is it hard to find such textbooks?

Professor Leader talking to Singapore teachers at the SMS Teacher's Workshop: How to mark hard question?

IL: I think it is hard to find such textbooks. Some do. For example, any book by Bela Bollobas has lots of thinking questions. Many books do, but many books don't. You mentioned statistics..very interesting. Normally in Cambridge, examples in statistics problem sheets are quite routine. But occasionally there are interesting thinking questions, and the students actually adore these, they really love these. Of course, on the exam, they don't like it so much. I'm not suggesting that on the exam paper, we have hard questions...that will be disastrous. But certainly on the homework assignments, questions students get to practise, it is really important that they get something to think, and not just apply, apply... Even questions which say: "Here is a theorem from lectures. In the theorem, you assume all random variables are positive. What if they can be negative?" Just a bit of thinking, instead of applying formula.

VB: Say I want to teach calculus this way. Would it be easy to think of a few such books?

IL: Yes, in some areas there are some nice books. If you want to do some kind of calculus at any level, books by Rudin have got some nice problems in them. It could be you are in some area where you invent your problems. Unfortunately it takes a lot of thought to teach a course. And if you really think hard about problems, it is very good.

VB: The main idea is, at least for students who do not intend to become professionals or routine users of mathematics, they should get from a university education some thinking ability, and this is best fostered through a teaching model that challenges them with questions.

IL: It is really important, because when people (software companies) are hiring maths graduates, they really need to be able to think and adaptable and can show learning skills. And for this, it is really important that the maths graduates don't just switch their brains off and follow algorithms, but can think. Often some of the courses are so hard and abstract, for the student to even understand the lecture notes and theorems, they have to work very hard and think. It doesn't happen automatically, but can be helped by making thinking questions. And it's very much the same that if you carry on to a maths PhD, it is unbelievably important they learn about thinking questions, and they learn about failure. PhDs, obviously, solve problems that no one solves yet, so it is very important that they use the idea.

VB: When you spend more energy building these very useful thinking skills, you will need to give up on some content. Is that true?

IL: If you teach a course, you teach the theorems, the examples, the proofs, as per normal. But in the homework assignments, that's where you should ask students some questions that need some thought. I think during the lecture, of course you can't...students cannot think in the lectures. During the lectures, students are copying, taking notes, maybe half asleep. Maybe they are totally lost. That's normal. In the lecture, you present the material. But then, the point of the homework sheets, or example sheets, should be to make the students partly understand the material, but much more to think about the material. Both are really important.

VB: In your mind, it is quite unrealistic to expect students to actually think during the lectures.

IL: I think it is impossible. Maths is hard. Almost every student in almost every lecture is lost after 15 minutes. Maybe they have a vague idea of what is going on, but the details are left out, even for the best students. It's human nature...most of the time they are not following it.

VB: So that happens to the high ability students?

IL: Absolutely yes. In Cambridge it is very very common. Perhaps, in a first year course, where the students already know the material, it's OK. But for new material, because maths goes fast and builds up on previous work as well, normally the best students are also totally lost. Sure. But they take the notes, and they go away afterwards and study hard. It's very important.

VB: This reminds me of experience as an undergraduate student in pure maths.

IL: I should say that I've talked to many students. A mediocre student is lost after 15 minutes. But also, a mediocre student looking around the room thinks "I'm the only person not following."

VB: Haha. In Cambridge, how often does a maths course involve students reading and learning from a textbook?

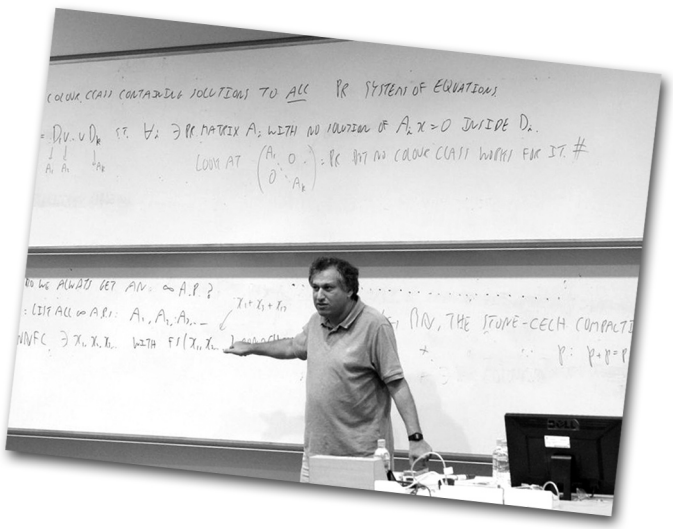
IL: Never ever. So in Cambridge, the maths course is based on the lecture. So the feeling is, if a student has copied out from the board, he will understand it. With a textbook, a student can read it line by line, but not understand in any depth. Also, if something is lectured on a blackboard, or whiteboard, there is a limit on how much you put on. So it stops overload. If it's a textbook, the lecturer says "Please read these 50 pages." and there is no way of knowing how hard this is for the students to do. The students might sometimes prefer a textbook, some background information, worked examples. Of course, there are many good textbooks. But in Cambridge, the lecture is all blackboard. In fact there is a great phrase in the Cambridge book which gives syllabus for each course, which says: "These syllabuses are maximal for examining and minimal for lecture." It means what is examined must be part of what is written down here, and nothing else, and the lecturer must cover what is down here.

VB: Do students ask for copies of the lecture notes, before or after the lecture?

IL: That is an interesting question. I think lecture notes with one exception are a very bad thing. So, roughly speaking --- I will get to the exception in a minute --- if the student has lecture notes, the student will not concentrate during the lecture. They will daydream, they will say "Oh I could read it some time." That's one thing. Second thing: the lecture gets much worse. The lecturer thinks "I can be disorganised. I can skip some bits which are in the notes." This is bad. And thirdly,

which is the most interesting one, we find when a student has written down stuff in his own handwriting, even if he doesn't understand it, he has some ownership that he wants to learn it. If a student photocopies other students' notes or gets printed notes, he doesn't own it, will never learn it. Interestingly, somebody noticed the following, that a lecturer for some course is much loved, then one year, the same lecturer, the same course, the students were much worse in the material, and it always turns out this was the year the lecturer gave out printed lecture notes. This is really quite remarkable. I think printed notes are a great great evil. All students want them, and think "Of course they help. How can they not help you?" But they are very bad, for these reasons. The one exception is, in Cambridge: when there is lecturer who is confused, and not a good lecturer...very unclear, just a bad lecturer. Then, sometimes if the students have access to previous lecturer's printed notes, that can help them. So I think printed notes are OK to save students from a bad lecturer, but in general, if it's a moderate or good lecturer, then they are really a great great evil.

VB: Would you extrapolate that statement to the general undergraduate student bodies, like in Singapore?



Prof Leader delivering an SMS Academic Talk: Partition Regular Equations at Nanyang Technological University.

IL: It's true in Britain. I can't see how it could not be true. If students write something down, they own it. With printed notes, they know they can some time or other read it, so they lose the impetus to read it. Also, such a lecturer covers everything. With printed notes, the lecturer can think "Oh I can give a sketch or an outline." Very very often, what happens is, the lecturer somehow subconsciously assumes the students have read the printed notes. So a lecturer says "In this theorem here, you will notice in the printed notes, the proof mentions induction. This is important because ..." That's great if the students have already, in advance, read the day's lecture notes. But of course real life isn't like that. Students never read beforehand. It's always a disaster. It's just human nature.

VB: That point about writing it and owning it is quite interesting. It's almost like sometimes, in order to remember something, I write it down and it helps.

IL: That's exactly right. Write it down, it's part of you. Maybe it's in your brain. It's active, and it parks something in your brain. Partly also it's your handwriting, you want to do something with it somehow. Even when you come to revise, you have forgotten it, people seem happier to revise from their hand-written notes than just some printed notes, which are anonymous.

VB: I heard there is a copy of Principia Mathematica in the Wren Library with Newton's handwriting?

IL: Yes, there is a first edition Principia, with Newton's own handwritten corrections...very impressive. It's right next to Newton's letter to Hooke about gravity, and some strange things. Newton's walking stick, a piece of Newton's hair, his watch, and most amazing, his little pocket book, which he wrote things down like "Ink 4 shillings". That's his expense book.

VB: Should students be encouraged to do expository writing in maths? If so, when is a good age to start?

IL: That's a very interesting question. I think it's good for the students to learn some writing skills, because even the very best maths students are normally incredibly bad at writing. Not only they can't write English sentences, but also they have difficulty dealing with the keywords, even maths proof words like "therefore", "so", "but" and "assume". It is so important to have the words.

VB: Even a mathematical proof can be quite sloppy?

IL: Yes, very sloppy. Many of our first year students, we spend a long time saying to them, "Yes, I know you can prove it, but what you have written is rubbish." So in that sense, anything they know already about this will be brilliant, it will help train them a lot. Should it be at school...I don't know. I imagine school curriculum is very crowded. If they have space to do some expository writing, maybe they can write a project, a write-up. If there is space for it, it is a very nice skill. Also, in later life, if they don't do maths, to be able to explain something is an important skill.

VB: In Cambridge, how is the writing skill instilled? Is it through students taking other courses? Or is there something special in the maths programme?

IL: Cambridge is very lucky to have what we call the supervision system. So students go to lectures, two hundred of them. They are likely to copy the lecture notes. Lecturer gives out a problem sheet, sheet of exercises. The students will do the exercise, then once a week the students have a supervision...that means two students to one professor, for an hour, each course. So it's very intensive. And during that, the supervisor looks at the students' work, reads it carefully, probably in advance, and says something like "This is badly written. Here's how you can improve it." So it's done by talking to the students. But that's a luxury tutorial system. If you are in most universities, each class has 6 or 10 people. Then, I don't really know how you would teach it. Unless you can find the time to get someone to read what the student has written. I don't believe in a separate expository writing class. I much rather they wrote the actual maths as how they learnt it. Imagine with that, the question of how you teach students elementary logic, the fact that "A implies B" is different from "B implies A". So my personal feeling is that teaching students standard logic is a waste of time. If you try to teach students how to negate a statement, you give them a statement "All pigs can fly." You teach them that then negation says "Some pig can't fly." So, they get very good at that, but still when they get to maths, they flip back and do things directly, in their own, wrong way. So I think the best place to teach elementary logic is during the supervision, when they first have a maths theorem to prove, like "A implies B". And they say "Suppose A isn't true. Then B isn't true and I'm done." and then you correct it. I quite firmly believe that teaching logic separately is useless, as they never learn to apply it in actual maths contexts.

VB: So the context, the problem, is important, to learn such basic skills. The NUS Faculty of Science has just started this semester an initiative, where it is compulsory for students, including maths and stats students, to enroll in a writing course.

IL: But meaning writing on some science, or in general?

VB: It is major-specific. So they read about things according to their majors.

IL: That's good. If it's reading about maths, then that's a good thing. Sure.

VB: The teacher is from the Centre for English Language and Communication, not a mathematician. We are trying it out.

IL: It will be interesting.

VB: You represented the UK in the International Mathematics Olympiad. May I ask, for the benefit of students, how that experience shaped your mathematical life? Did it change things a lot?

IL: It actually changed things a lot. Of course, I really loved maths before that. In those days, there wasn't really any training. We didn't meet other students who were good. So in fact, you met the rest of the team for the first time at the airport going to the IMO. From Cambridge, I knew several other people who loved maths, who were my friends already, so I wasn't starting as a lone mathematician. I thought that was very nice. Actually, on a positive level, some of my absolutely closest and dearest friends are people whom I met at the Olympiad. So a lot of the British team became my closest and best friends for life. For me, personally, it was a really wonderfully enriching thing. I would be a much poorer person if I hadn't met those friends.

VB: Did it have much impact on you becoming a professional mathematician?

IL: No, I don't think so. I always loved maths, always wanted to come to Trinity and do maths. Of course, I always wanted to be in the IMO team. If I hadn't been in the team, I would have been sad about that, but I would still be doing maths.

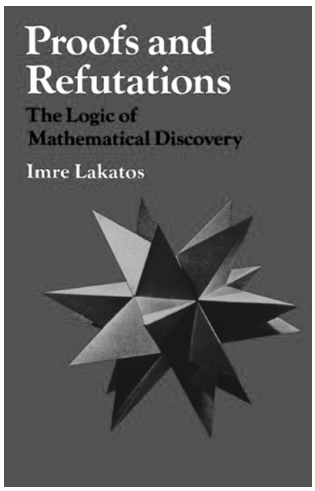
VB: At that age, did you know that you were going to be a professor of mathematics?

IL: I knew I wanted to do maths. So this is the way I view maths, often there is no decision to make. So you do maths in university because you like maths...no decision. If you like, you go on to PhD, not because you've carefully weighed up the options and decided "Oh let's do a maths PhD." And then you do get that, you try for a postdoc or something. Again, not because it's a choice of "Shall I try a postdoc or not?" but a natural progression. So it's a natural progression which I think that, along with many others mathematicians, I wanted at a very young age, without consciously thinking about it. Not about a career goal at all.

VB: That's kind of nice.

IL: Yes, it's nice not to have the angst and the worry of "What shall I do next?" It's a very lucky position to be in.

VB: Your first name is very interesting. You were a godson of Imre Lakatos. Can you tell me something about him? Did you interact much with him? How did he affect your life?



IL: He was a famous Hungarian philosopher of mathematics, a very interesting man. My mother is Hungarian, so she knew him already, and he was a good family friend, that's how he became my godfather. I knew him pretty well. I was only about 10 when he died. I remember especially he bought my first Monopoly set. So I loved him as a friend then. When he was alive, I never had a maths discussion with him. Since then, I have read and enjoyed his famous book "Proofs and Refutations".

VB: I, too, enjoyed this book. The book says something to the effect that what is presented has some implication for mathematics education. What do you think about that? Has anyone written some textbooks that follow his philosophy of presenting maths.

IL: I don't know the answer to that. It's certainly a very nice way of approaching problems. The basic idea is you should try to understand some phenomenon. You look at examples. You form your conjecture. And then, it's very important you spend a lot of time looking for counterexamples. That's his basic idea. It's incredibly important. If you don't spend your time looking for counterexamples, you may waste your time trying to prove a thing that isn't true. So it's very important to look for counterexamples. A great saying of my supervisor Bela Bollobas: "If you're trying a question where the answer is maybe it's always true, maybe there is a counterexample. Whichever one you think, you should always try to find a counterexample because if you find a counterexample -- great, it's done. If you don't find one, hopefully understanding why you couldn't find one would begin to bring you to the proof. So the idea of counterexamples, even if it's a true theorem, the idea that you start by looking for a counterexample, is very important. Even if the question says prove that every A has property B. It's very important to first try to find an example of A that doesn't have property B. Of course you will fail, but in failing you should begin to understand what makes it true. So many students are very keen to sit at home and try to prove B but without insight or a gut feeling. They have zero chance of success.

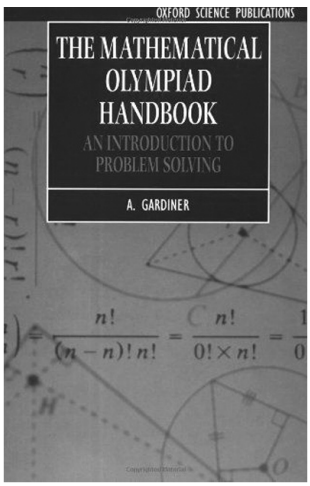
VB: That is very interesting. So you would say that it's very important for mathematicians to even try to prove a false result?

IL: Absolutely. Most mathematicians know the following: If you are reading a book and it says "Here is the proof", you should put the book down and try to prove it yourselves. That's pretty common knowledge. But this is saying, in addition, to prove it yourself, the first step is to try to make it false. Of course you will fail, but hopefully that will then tell you why it's true.

VB: That's interesting. You also mentioned you don't think there is any textbook that is written in this historical perspective.

IL: I don't know. There might be. There isn't one that I know about. Certainly in the Polya's book on "how to prove it", there are several sections that have the same spirit and idea.

VB: Polya's book is very famous.



IL: And a very good book. In terms of textbooks which help you think of how to prove things, for school students there is one brilliant book by Tony Gardner called "The Math Olympiad Handbook". The reason this book is so exceptional is it tells you how to approach a problem which you can't solve straightaway. So there are many, many excellent books that tell you "here are some new techniques to learn, from inequality techniques and induction, this and that, but they don't tell you how to conquer your fear of the unknown. Tony Gardner's book tells you how to conquer your fear of the unknown. It says "Experiment, try some examples". So in his book, you'll see, problem, solution and the solution begins with "What should we do if we are scared?" So it's a very good book. That book is putting into practice the philosophy that before you prove something, you must try to get a feel for it, or else you have approximately zero chance of solving it.

VB: Sounds to me that that book is a great book for learning how to become a researcher in mathematics. Our conclusion here is "It's very hard to learn how to be a professional by looking at a clean, logically presented textbook that says definition, theorem, proof,...

IL: Absolutely. If you are looking at proofs from lectures, you should read the theorem and put your notes down for a second, cover it up, and try to prove it yourself. Of course, probably you will fail,…if it's a hard theorem. At least you'll see why you fail and then you look at the proof and you go "Ah, the key idea is just said there". Of course, I should say that's in a dream world where students have lots of time. Probably in real life, students are quite busy. If they have time, it will be an excellent way to use their time.

VB: At some point, professional mathematicians, in their training, they have to go through this phase somewhere. Probably in their PhD phase and not so likely in their undergraduate days, except for some of the more precocious ones. Would you say that?

IL: Right. Many many students unfortunately are rarely the thinking type. They should be but they aren't because they are not interested in the problems. Maybe the first time they think is during their PhD. Maybe the first time they learn, their PhD supervisor gives them a problem. Maybe they stick it out for months but they just get nowhere on it. They have to learn how to think about things. You don't just have problems and instant solutions.

VB: On behalf of the SMS, I thank you very much for the interview.

Welcome dinner with SMS Exco members.
Standing from left: Chew Tuan Seng, Wang Haibin, Toh Pee Choon, Yap Von Bing (interviewer of this article), Ng Kah Loon, Victor Tan;
Sitting from left: Hang Kim Hoo, Ling San (SMS President), Prof Leader, Toh Kim Chuan.