Category A (Lower Secondary) winner

Is God a Mathematician?

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Stop for a second. Let your surroundings grind to a halt. Take a deep breath, and look at the world around you. Whether you are drinking a cup of frothy coffee, admiring some flowers or playing basketball with some friends, stop for a while and marvel at the mathematics behind seemingly un-mathematical activities. Did you notice the angles in the coffee foam? The parabola of the basketball's arc? The spirals in a flower's pollen?

Why is mathematics so unreasonably effective? Virtually everything on this earth, be it natural or man-made, is somehow related to mathematics in one way or another.

Why? Is God a mathematician?

Let's find out.

Close your eyes, and imagine yourself in a garden- a beautiful garden with flowers, bees, ponds, and butterflies. You walk over to the pond, and toss a stone in it, gazing at the ripples that vibrate across the pond. Some water also splashes out. Did you know that this splash is actually almost symmetrical?

This is an example of radial symmetry occurring when several cutting planes of an object produce roughly identical pieces. This can be exhibited in many examples. One of them is the petals of flowers. Most plant cells are radially symmetric due to the way in which they grow. The petals, sepals and even stamen are radially symmetrical! Sessile, unmovable animals such as the sea anemone and corals share such characteristics. Floating and slow moving animals like the jellyfish are radially symmetrical as well.

Wow! You shake your head in amazement. Who knew that even a simple water ripple had so much math explaining it?

A bee buzzes around you, and you follow the beating sound of his wings to a beehive. You notice a small hole and from a safe distance afar, you peer in and gasp when you see the honeycombs.

Rows after rows of precise hexagonal wax cells line the beehive, forming beautiful patterns. God gave bees the skills to construct such nests to store pollen, honey and their young. Then why is the term honeycomb used to describe tessellations as well? The very reason behind this is that honeycombs themselves are actually natural tessellations! A normal honeycomb is made of many hexagonal shaped grids. They hold the comb together and prevent it from being too delicate. Also, the hexagonal grid of wax cells on either side of the nest are slightly offset from each other. This increases the strength of the comb and reduces the amount of wax required to produce a more robust structure.

You whistle in admiration of the product of the bees' hard work. Suddenly, though, it starts to snow and you hurry over to a small pavilion in the midst of some palm trees and several sunflowers. Reaching out, you catch a snowflake in your hand, and its astounding beauty takes your breath away. They all look the same... Or are they? The truth is, God gave every snowflake its own unique pattern! There is more to these charming ice crystals than meets the eye. God made every single snowflake that would fall down from the sky different from its counterparts, just like He would make any human; no two are alike. Probably in a bid to make the intricate flakes look even more exaggerated, He even ensured that every one of them had at least one line of symmetry.

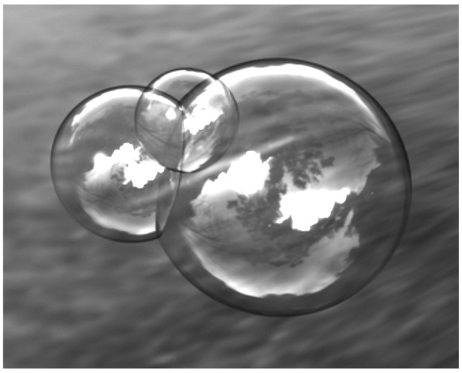
Snowflakes are basically transparent ice crystals. They appear white due to refraction of light. Snowflakes begin as snow crystals which form when microscopic super-cooled cloud droplets freeze, and the frozen droplets fall through the atmosphere of the Earth. It is unlikely that any two snowflakes are alike since the water molecules which make up a typical snowflake grow at different rates and in different patterns depending on the changing temperature and humidity within the atmosphere. Despite claiming that snowflakes are symmetrical, so far, studies show that only 0.1% of snowflakes exhibit the ideal six-fold symmetric shape. This is because, even though the conditions around the snowflake are almost identical, it does not guarantee that all the arms of the snowflake grow exactly the same way.

How nice of God, you think, to make nature so intricately beautiful. You sit down on one of the stone benches, and pluck a pinecone from a nearby tree. It suddenly hits you- pinecones can do math too! They are good at Fibonacci numbers. All cones grow in spirals, starting from the base where the stalk was, and going round and round the sides until they reach the top. There are two sets of spirals for each pine cone, going different directions. For all pine cones, the number of spirals in the two directions are next-door Fibonacci numbers. The smallest pine cone above has three spirals in one direction and five in the other. The medium one has five in one direction and eight in the other. It's not just pine cones. Sunflower seeds grow in similar spirals and so do most plants, again in pairs of Fibonacci numbers. There is a reason for this. Fibonacci numbers are an approximation to an irrational number which means that the seeds will not line up with each other, which could weaken the flower head or pine cone.

You laugh in your head- plants doing math! Surely God must have a great sense of humour in doing this. You look out of the pavilion and realise that the snow has stopped. However, a sprinkler system pipe has malfunctioned and is foaming.

Foam. Bubbly and white. Surely there is no math involved?

There is. Joseph Plateau established the basic geometrical and topological laws of foam equilibrium. In nature, the foam bubbles, composed of soap films, obey Plateau's law, which requires three soap films to meet at each edge at 120 degrees and four soap edges to meet at each vertex at an angle of about 109.5 degrees. The foam bubbles are made up of a junction, a Plateau border, and soap films. Foams may also be dry or wet.



Our knowledge of foams has helped the polyurethane industry and chemical engineering for viscous effects in ordinary liquid foams. Foam patterns which obey Plateau's law are also common in living cells; radiolarians and sponge spicules all resemble mineral casts of Plateau foam boundaries. The surface tension of the foams also affect the stability of the soap films, which accounts for the total liquid content, which affects properties such as conductivity. Hence, this may have some uses in the industry. Foams have also greatly improved our understanding of liquid drainage. The current generation of foam experiments aims to study the drainage and rheology of wet foams and the formation and collapse of metallic forms in the absence of gravity.

You are wowed by the number of math facts you have learnt today- all just from a short imaginary walk around a garden. From foams to pine cones, ripples to snowflakes- all of these represent math concepts. Or do math concepts represent them?

Nobody knows, except God, the omnipotent, omnipresent person. So, here comes the big question: is God a mathematician?

We'll leave you to find out.

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Category B (Upper Secondary) winner

3D Model Restoration from 2D Satellite Pictures by Spherical Geometry

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Introduction

Satellites are widely used nowadays. In the case of visible-light satellite pictures, the surface of the Earth is projected to a 2D image. During this projection, information such as a uniform scale is lost. For example, points that are 1cm apart in the picture may correspond to varied distances due to the projection. Thus, we cannot get the distance between two points or geographic coordinates of points by direct measurement.

In this essay, we found a way to rebuild the detected part of the 3D Earth surface from any 2D images by finding the geographic coordinates of any point in the picture.

The first step is to find the relationship between apparent and actual radial distances of an arbitrary point in the picture. Then, the problem is solved within two cases:

- geographic coordinates and north direction of the center of picture are given;
- geographic coordinates of any two points in the picture are given.

The first case is a special case of the second (equivalent to given the center and the North Pole). This indicates that we are able to find the geographic coordinates of any point in a picture with two other points given (in an ideal situation[1]).

The essential mathematical tool used in the essay is Spherical Geometry. A brief summary of terms and rules in Spherical Geometry are given in the Appendix. In this essay, "coordinates" always refers to geographic coordinates. All coordinates and angles are in degrees. Range of latitude is $[-90°, 90°]$, where positive latitudes represent the northern hemisphere. Range of longitude is $(-180°, 180°]$, where positive longitudes represent the eastern hemisphere. O shall denote the centre of the spherical Earth and r shall denote the radius of the Earth.

Finding the Actual Radial Distance of a Point

Figure 1 below represents a 2D satellite picture at the orbit directly above the Earth, in which X' is the centre of the circle, A' is an arbitrary point and G' is the intersection of line $X'A'$ and the circle. Lengths of $A'X'$ and $G'X'$ in the picture are m_1, m_2 respectively.

[1] Idealization is discussed in the last section.

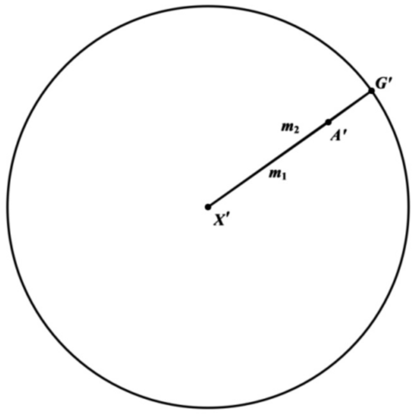


Figure 1. A satellite picture

Figure 2 below shows the cross-sectional plane along $X'G'$ and the satellite, where the leftmost dotted line represents the cross-section of the picture and the circle represents the cross-section of the Earth surface; M is the lens of the camera on the satellite, and its distance MX from the Earth is known; X, Q, T are actual locations represented by X', A', G' respectively in the picture; A, G are the projections of Q, T on the plane parallel to the picture across X; the picture is essentially a scaled image of this plane; $\varphi = \angle TOX$; $\theta = \angle QOX$; and $TF \perp XO$; $QR \perp XO$.

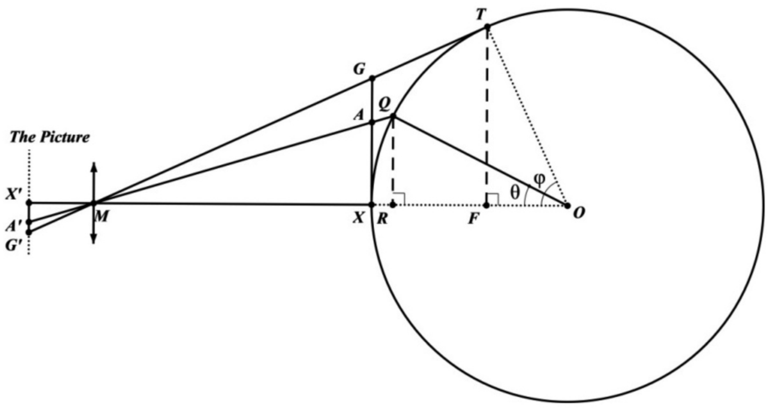


Figure 2. The cross-sectional plane along X′G′

Our task is to find the actual radial distance of Q from X, i.e., the arc length of XQ. Since G' is a point on the outline of the picture, MT is the tangent to the great circle. Therefore, MT can be found by

$$MT^2 = MX \cdot (MX + 2r)$$

using Pythagoras Theorem. Then, the central angle φ can be found by

$$\varphi = \arctan\left(\frac{MT}{r}\right).$$

As ΔTMF and ΔGMX are similar,

$$\frac{GX}{MX} = \frac{TF}{MF} = \frac{r \cdot \sin\varphi}{MX + r - r \cdot \cos\varphi}.$$

Therefore,

$$GX = \frac{MX \cdot r \cdot \sin\varphi}{MX + r - r \cdot \cos\varphi}.$$

Since the satellite picture is a scaled image of the plane XAG, AX and $\angle QMO$ can be found by

$$AX = \frac{GX}{G'X'} \cdot A'X' \quad \text{and} \quad \angle QMO = \arctan\left(\frac{AX}{MX}\right).$$

The angle $\angle MQO$ can be found by applying sine rule to the ΔOMQ,

$$\frac{r}{\sin \angle QMO} = \frac{r + MX}{\sin \angle MQO}$$

and then the central angle θ and arc length XQ can be easily found from here.

3D Model Restoration from 2D Satellite Pictures

From now on, we shall denote points on the surface of earth and their corresponding points in the satellite picture using the same letters.

Geographic Coordinates and North Direction of the Centre are Given

As shown in Figure 3 below, point P is the central point with given latitude and longitude (m, n) where $m \in (-90°, 90°)$[2] and $n \in (-180°, 180°]$. The dotted line is the meridian on which P lies and the arrow points to north. An arbitrary point Q is chosen. The task is to find the latitude and longitude of the point $Q(x, y)$.

In the picture, we can measure the apparent distance of PQ and the angle $\angle QPC = \alpha$.

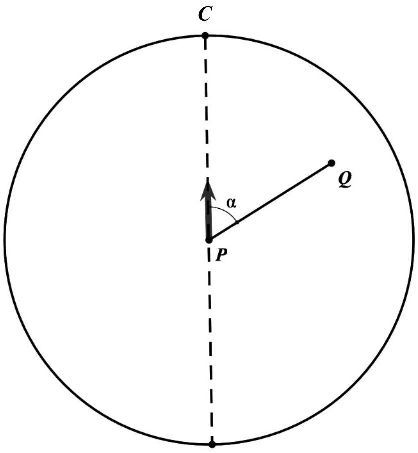


Figure 3. A satellite picture

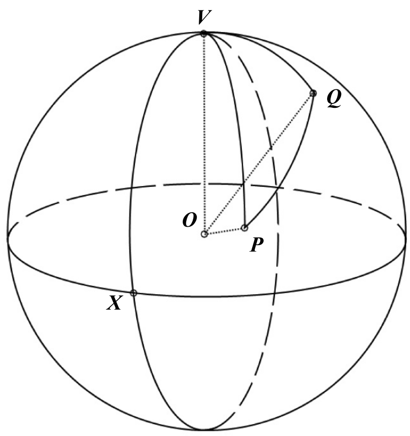


Figure 4. The surface of the Earth

In Figure 4 above, we introduce the North Pole V (it may not be in the satellite picture) so that VP is a segment of the meridian passing through P, and $\angle POQ$ can be found as given by the previous section, and

$$\angle VOP = 90° - m \quad \text{and} \quad \angle QPV = \alpha.$$

Using the cosine rule for spherical triangle ΔPQV[3], $\angle VOQ$ can be found from

$$\cos \angle VOQ = \cos \angle VOP \cdot \cos \angle POQ + \sin \angle VOP \cdot \sin \angle POQ \cdot \cos \angle QPV.$$

Since VQ is the segment of meridian from North Pole to point Q, the latitude of point Q is calculated by $x = 90° - \angle VOQ$.

For spherical triangle ΔPQV, we can use the sine rule to find $\angle PVQ$,

[2] If P is North/South Pole, we need to know the longitude of the dotted line (which can be any line passing through P in this case). Longitude of Q is that longitude adding/subtracting α accordingly and latitude of Q is $\pm\frac{PQ}{r} \cdot 90°$, where the sign is positive if P is North Pole and negative if P is South Pole.

[3] If P, Q, V are collinear, it is considered as a triangle with two 0° angles and a 180° angle.

$$\frac{\sin \angle PVQ}{\sin \angle POQ} = \frac{\sin \angle QPV}{\sin \angle VOQ}.$$

Since VX is a segment of Prime Meridian, and VQ is a segment of a meridian, the longitude y of Q can be calculated by $\angle PVQ$ and the longitude of P.

Therefore, we have obtained the latitude and longitude of point $Q(x, y)$ in this picture.[4]

Geographic Coordinates of any Two Points in the Picture are Given

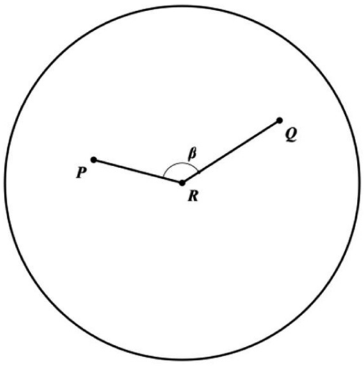


Figure 5. A satellite picture

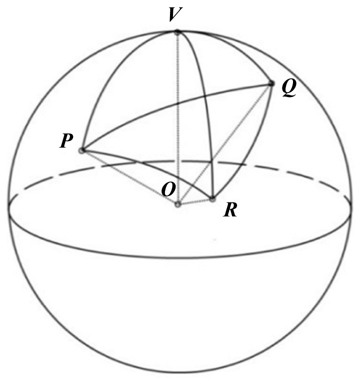


Figure 6. The surface of the Earth

For any two arbitrary points P and Q in the picture with their coordinates given, we are going to find the coordinates of any other point in the picture as shown in Figure 5.

There are two steps in this case:
Step 1: Use the geographic coordinates of P and Q to determine the coordinates of the center R of the circle.
Step 2: Find the geographic coordinates of any other points by the coordinates of P and R.

Same as previous section, there are other possible positions for P, Q, and R. Here, we just use this example below to illustrate the algorithm and method. The method is almost the same for other cases.

Step 1: find the coordinates of R

We introduce the North Pole V in Figure 6 and two spherical triangles are formed, ΔVPQ and ΔPQR. To find the latitude and longitude of point $R(x, y)$ in this picture, we just need to find the two angles $\angle VOR$ and $\angle PVR$ respectively.

The degree of $\angle PRQ$ can be measured from the picture. As in the previous section, we are able to obtain the degrees of $\angle ROP$ and $\angle ROQ$, and so the lengths of PR and QR.

First, we apply cosine rule to ΔPRQ, we can find $\angle POQ$ by

$$\cos \angle POQ = \cos \angle ROP \cdot \cos \angle ROQ + \sin \angle ROP \cdot \sin \angle ROQ \cdot \cos \angle PRQ$$

[4] Here, the picture is used to illustrate the method. In the actual case, the formula may be slightly different depending on the position of P and Q.

Then we apply sine rule to ΔPVQ and ΔPRQ, we can obtain $\angle VPQ$ and $\angle RPQ$ from

$$\frac{\sin\angle VPQ}{\sin\angle VOQ}=\frac{\sin\angle PVQ}{\sin\angle POQ}\quad\text{and}\quad\frac{\sin\angle RPQ}{\sin\angle ROQ}=\frac{\sin\angle PRQ}{\sin\angle POQ}$$

respectively.

Therefore, we can find the angle $\angle VPR$ by using the degrees of $\angle RPQ$ and $\angle VPQ$, and so the required angle $\underline{\angle VOR}$ by applying cosine rule to ΔVPR:

$$\cos\angle VOR=\cos\angle ROP\cdot\cos\angle VOP+\sin\angle ROP\cdot\sin\angle VOP\cdot\cos\angle VPR,$$

and the degree of $\underline{\angle PVR}$ can be found by using sine rule to the same triangle ΔVPR:

$$\frac{\sin\angle RVP}{\sin\angle ROP}=\frac{\sin\angle RPV}{\sin\angle VOR}.$$

Step 2: find the coordinates of any point K

Now we find the coordinates of another point $K(x_k, y_k)$ in Figure below with coordinates of the center R and another point P. Similarly, we just need to find angle $\angle VOK$ and $\angle RVK$.

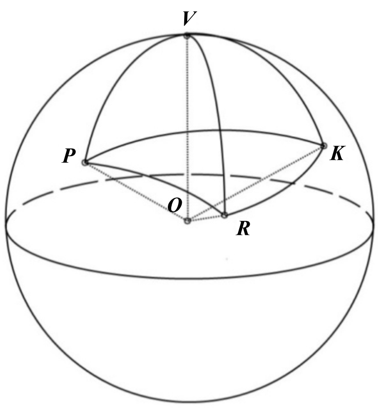


Figure 7.

First, $\angle PRV$ can be simply calculated by sine rule to ΔVPR:

$$\frac{\sin\angle PRV}{\sin\angle POV}=\frac{\sin\angle PVR}{\sin\angle POR}.$$

And then, $\angle PRK$ can be directly gotton from the satellite picture, and we get,

$$\angle VRK=|\angle PRK-\angle PRV|.$$

Therefore, applying cosine rule to ΔVRK, we obtain the required $\underline{\angle VOK}$ from

$$\cos\angle VOK=\cos\angle VOR\cdot\cos\angle ROK+\sin\angle VOR\cdot\sin\angle ROK\cdot\cos\angle VRK$$

Also we can apply sine rule to ΔVRK, and it gives the required $\underline{\angle RVK}$ from

$$\frac{\sin\angle RVK}{\sin\angle ROK}=\frac{\sin\angle VRK}{\sin\angle VOK}.$$

NOTE: All the measurement from the satellite picture require R being the center of the circle; otherwise, the connection of two points is on a great circle.

Limitations and Further Improvements

We assume that the Earth will always appear as a 2D circle on a picture, and this is how we find the accurate scale of a picture. If the range of photo-taking is limited such that the circumference of the Earth is not shown on the picture, the scale of the satellite picture is required.

For ease of computation, we assume the Earth is a perfect sphere with uniform radius at any point of its surface. This makes the scale less accurate. To achieve greater accuracy, ellipsoid is a better model. A 3D model based on geodesic data is an even better one, but the calculation is much more complicated.

We assume that during the process of satellite taking photo, the signal is not by any means interfered, so noise is perfectly absent in the picture. This affects the accuracy. Harmonic analysis and wavelet analysis can be used to minimize the effect of noise.

We assume that the camera points toward the Earth, so the imaging plane is perpendicular to the line linking center of the Earth and the camera (modeled as a point). However, camera may take picture with a significant angle, so further analysis that are similar to what we have done in this essay is necessary as the tilt will significantly change the relation from spherical surface to planar picture.

Conclusion

Under the assumptions above, we are now able to determine the geographic coordinates of any point in a satellite picture with any other two points in the same satellite picture. This method of determining geographic coordinates of a point in picture can be used to interpret the satellite pictures better.

Appendix: Spherical Geometry

The difference between Spherical Geometry and Euclidean Geometry

The fifth postulate in Euclidean Geometry says that, [1]

> *If a straight-line falling across two (other) straight-lines makes internal angles on the same side (of itself whose sum is) less than two right-angles, then, being produced to infinity, the two (other) straight-lines meet on that side (of the original straight-line) that the (sum of the internal angles) is less than two right-angles (and do not meet on the other side).*

However, in Spherical Geometry, if two lines both perpendicular to a third line, and are extended infinitely, they will eventually meet instead of being a pair of parallel lines (such as two longitudes on the surface of the Earth). This major difference leads to many other differences in the properties of basic geometric elements such as triangles and straight lines.

Definition of basic elements in Spherical Geometry [2]

Straight Line: A section by a plane passing through the center of the sphere

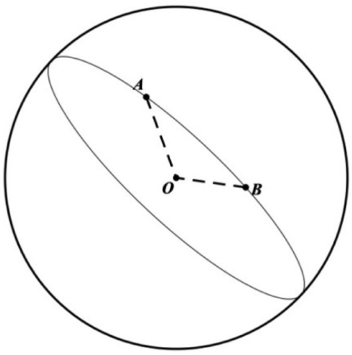
Spherical Angle: The angle formed by two intersecting arcs of great circles.

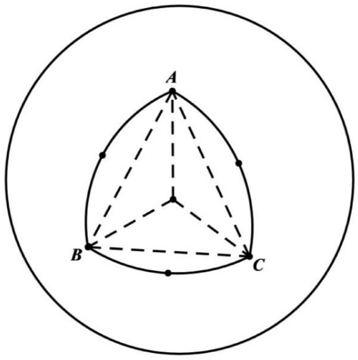
Triangle: A portion of the surface of a sphere which is bounded by three arcs of great circles.

Central Angle: An angle whose vertex is at the center and whose sides are radii.

Calculating the length of segment

The length of a segment, basically part of a great circle, is calculated in the same way as calculating the length of arc of circle. In the left diagram below, we can see that $AB = r \times \angle AOB$.





Sine Rule and Cosine Rule in Spherical Geometry [3]

In the right diagram above, we denote the angle $\angle BOC$, $\angle AOC$, $\angle AOB$ as a, b, c,, and angles $\angle CAB$, $\angle ABC$, $\angle BCA$ as A, B, C respectively.

Then, Sine Rule in Spherical Geometry can be expressed as:

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

and Cosine Rule in Spherical Geometry can be expressed as:

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$
$$\cos b = \cos c \cdot \cos a + \sin c \cdot \sin a \cdot \cos B$$
$$\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos C$$

Bibliography

[1] Fitzparick, R. (2007). *Euclid's Elements*.

[2] Hull, G.W. (1897). *Elements of Geometry: including Plane, Solid and Spherical Geometry*. E.H. Butler & Company.

[3] Todhunter, I. (1863). *Spherical Trigonometry: For the Use of Colleges and Schools*. Macmillan and Company.

Category C (Junior College) winner

The Mathematical Earth

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Mathematics has been essential in human lives since the prehistoric era. The need for counting gave birth to arithmetic, and Mathematics gradually established its role as a tremendously useful tool for human.

Plato, the founder of the first mathematical centre (Platonic Academy), created his Theory of Ideals with Mathematical objects as his most prominent examples. Mathematics is placed in the ideal world, since a real-life straight line can never be straight and thin enough, but a mathematical line is perfectly straight and one-dimensional. Lacking physical extension, mathematical entities seem to be separated from our reality, from nature, from Earth. The distinct nature of the two worlds implies a lack of connection between Mathematics and Earth, which, arguably, contradicts the evidence we have on the extensive applications of Mathematics in natural and human processes on Earth.

It has been generally accepted since the last millennium that there exists a connection between Mathematics and nature, particularly planet Earth. The heavenly bodies seem to possess some Mathematical order, and so does Earth. Kepler and Newton successfully described planetary motions and gravitational force using Mathematical relations[i]. Fluid motion, from surface waves to deep currents, is investigated with the theoretical basis of partial differential equations, particularly the Navier-Stokes equations which approximate the behaviour of marine processes. If Mathematics belongs to the world of ideals as Plato suggested, it must be a miraculous coincidence that oceanography and astronomy are connected to mathematical analysis to such a large degree.

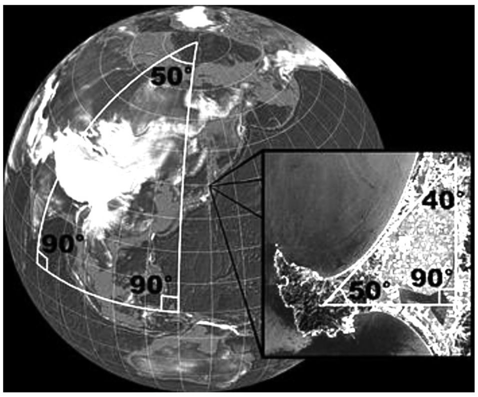

Figure 1: How Riemannian and Euclidean geometry differs on the surface of the Earth[ii]

Once we have established the existence of an inextricable relationship between Mathematics and Earth, it is worth noting that this connection is two-way: The Earth can alter the development of Mathematics and vice versa. Euclidean geometry with the parallel postulate remained the only geometry system until modern cartographers realized that the surface of Earth did not match the intuitive notion of geometry and questioned the validity of this axiom. The Earth inspired humans to look at the internal consistency and coherence of the Euclidean system and propelled the creation of a new paradigm: Riemann geometry. This differential geometry with complex manifolds later enabled Einstein's theory of special relativity, which in turn further explained the anomalous perihelion precession of Mercury. This characteristic of the Earth prompted a new Mathematical paradigm to be developed, which contributed to a deeper understanding of nature. Conversely, Mathematics indirectly alters our planet by guiding human decisions. Statistical tools such as multivariate analysis help us optimize the outcomes given fixed parameters. Collective motions in animals are described by the Lagrangian models (for individuals' actions) and the Euler model (for population statistics), which are crucial to decision-making involving ecosystem. The relationship between Mathematics and Planet earth is indeed indubitable.

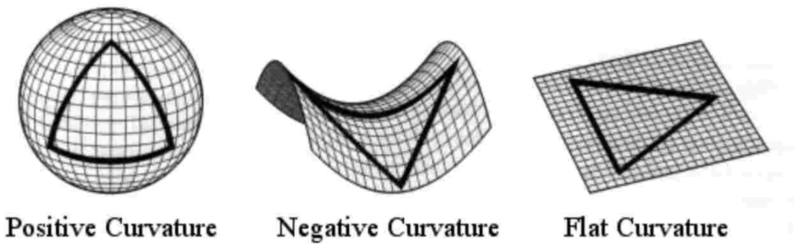

Figure 2: Riemannian geometry in relativity - an example[iii]

Another astonishing feature of Mathematics is its incredibly accurate description of our Planet. Computer models and automata theory are used to model enzyme kinetics and growth of cells[iv]. A. Fisher's[v] "Fundamental Theorem on Natural Selection"[vi] supported and supplemented Charles Darwin's Natural Selection theory. The principle of non-invasive scanning techniques CAT, MRI and PET rely on mathematical relationships. Even in the social sciences, Mathematics plays a significant roles. Matrix theory leads to the application of stochastic matrices and Markov chains in modelling changes in society, for instance, population growth, disease spread or genetic distribution. Number theory enables us with secure communication (asymmetric cryptography). Graph theory has proven its usefulness in modeling economic and social networks. A social structure without Mathematics is simply unimaginable and probably primitive and chaotic. Mathematics may be intrinsic not just in how nature operates, but also in how we organize our society.

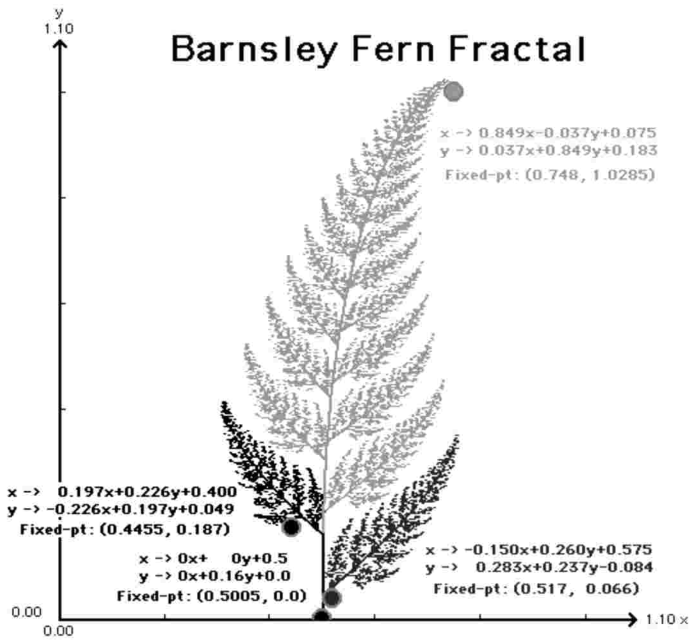

Figure 3: Barnsley's fern: an iterated function system that creates a fractal that matches the shape of a naturally occuring entity - a fern[vii]

It is rather puzzling why Mathematics can be so successfully applied in both the natural and social sciences. Is it possible that Mathematics is inherent in nature, and scientists when investigating the Earth are simply uncovering these Mathematical laws? One may be tempted to agree with the mathematical universe hypothesis: *Our external physical reality is a mathematical structure*.

However, there is another explanation for the miraculous success of Mathematics in describing our planet. Instead of accepting the intrinsic property of Mathematics in nature, it is entirely possible that humans are imposing our mathematical interpretation on nature. Mathematical Modeling, the process that has created the connection between Mathematics and the Sciences, involves making assumptions and bringing real-life situations to a simplified, solvable model. Humans, seeing Mathematics as an effective tool, might have been viewing all problems through a Mathematical lens. Just like how Kant argues that we cannot remove our subjective, innate way of looking at the world (including the notion of space-time and causation), we are not able to remove the lens of Mathematics from our perception of the Earth. Any relationship in nature can be modelled using mathematical equations, because assumptions have been taken to bridge the gap between reality and Mathematical models. Even if this is the case, there is no concern for giving mathematics such a special status. If Mathematics is really a lens that separates us from reality, past experiences have proven that this lens only zooms in on finer details and gives us more insights of the world, instead of tainting our vision. Whichever stand we take, whether Mathematics is inherent in nature or imposed on the Earth, Mathematics is undeniably an exquisite tool to describe, gain understanding of and predict nature.

Despite being the prime method of investigating our planet, Mathematics does not always give us the ultimate answer. There are always unknown factors in Mathematics and unpredictable phenomena on Earth that we do not yet understand. For instance, records of earthquakes reveal a linear trend between the magnitude of quakes and their frequency[viii], but do not predict where and when they occur. When using existing trend to predict future events, we are implicitly assuming that nature is uniform and predictable, and unfortunately, Mathematics cannot justify those assumptions. It is also questionable whether the mathematical foundation of quantum theory is sound, as the Yang-Mills problem is still one of Clay Institute's Millenium Problems. Both Mathematics and our planet are mysterious and never-ending puzzles that will continue to delight humans in our progress of expanding our store of knowledge.

References

[i] Kepler's equation of Elliptical Motion has given us an approximately accurate picture of the motion of the Earth around the Sun. The Earth's orbit can be modeled by fixed sets of equations. More information can be found at http://www.akiti.ca/KeplerEquation.html

[ii] Image retrieved in 2013 from http://upload.wikimedia.org/wikipedia/commons/thumb/9/97/Triangles_(spherical_geometry).jpg/350px-Triangles_(spherical_geometry).jpg

[iii] Image retrieved in 2013 from http://abyss.uoregon.edu/~js/images/universe_geometry.gif

[iv] This is a summary of the claims in the research paper extracted from http://www.ams.org/notices/201007/rtx100700851p.pdf

[v] *"The rate of increase in fitness of any organism at any time is equal to its genetic variance in fitness at that time."* (Fisher 1930), extracted from http://equation-of-the-month.blogspot.sg/2011/01/fishers-fundamental-theorem-on-natural.html

[vi] http://www.maa.org/mtc/Jungck-10-equations.pdf

[vii] Image retrieved in 2013 from http://www.faculty.umassd.edu/adam.hausknecht/temath/TEMATH2/About/Assets/GraphOfBarnsleyFernBig.gif

[viii] Information retrieved from http://earthquake.usgs.gov/earthquakes/eqarchives/year/eqstats.php