Mathematical Medley Problems Corner

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A. Prized Problems

1 . Does there exist a function $g : N \to N$ such that

$$g(g(m-1)) = g(m+1) - g(m)$$

for all natural numbers $m \geq 2$? Justify your answer.

2 Find all positive integers a, b and c such that

$$\frac{a^2+b^2}{3ab-1} = c.$$

[*Problem 2 was proposed by Albert F.S. Wong, Temasek Polytechnic.*]

B. Instruction

(1) Prizes in the form of book vouchers will be awarded to one or more received best solutions submitted by secondary school or junior college students in Singapore for each of these problems.

(2) To qualify, secondary school or junior college students must include their full name, home address, telephone number, the name of their school and the class they are in, together with their solutions.

(3) Solutions should be sent to : **The Editor, Mathematics Medley, c/o Department of Mathematics, National University of Singapore, 2 Science Drive 2, Singapore117543** ; and should arrive before 31 August 2008.

(4) The Editor's decision will be final and no correspondence will be entertained.

Problem 1.

Find all positive integers a, b, c, d, all of which between 1 and 9 inclusive, such that

$$\frac{1333a+130b}{20c+332d}=1.$$

(*Solution proposed by Dai Zhonghuan, Hwa Chong Institution Boarding School.*)

Given that a, b, c and d are positive integers satisfying

$$\frac{1333a+130b}{20c+332d}=1,$$

we let $k = 1333a + 130b = 20c + 332d$. Since $20c + 332d$ is an even number, a must also be even. Since

$$k = 20c + 332d \leq 20 \times 9 + 332 \times 9 = 3168,$$

a can only be 2. Furthermore, since 10 divides $130b - 20c$, we must have $1333a \equiv 332d \bmod 10$. Hence

$$332d \equiv 1333 \times 2 \equiv 2666 \bmod 10.$$

Thus, we have

$$2d \equiv 6 \bmod 10 \text{ or } d \equiv 3 \bmod 5.$$

Thus, as $1 \leq d \leq 9$, $d = 3$ or 8.

Using $332d = 2666 + 130b - 20c$ and the assumption $1 \leq c \leq 9$ and $1 \leq b \leq 9$, we conclude that

$$332d \geq 2666 + 130 - 180 = 2616.$$

Hence $d > 7.88$. As d is either 3 or 8, d must now be 8. So, $k = 2666 + 130b = 20c + 2656$, which simplifies to $1 + 13b = 2c$. Also, since $2 \leq 2c \leq 18$, we have $2 \leq 1 + 13b \leq 18$, which reduces to

$$\frac{1}{13} \leq b \leq \frac{17}{13}.$$

Thus, we conclude $b = 1$ and $c = 7$. Hence $a = 2, b = 1, c = 7$ and $d = 8$.

Editor's Note

Similar correct solutions were submitted by Wang Lu and Wang Shizhi from Hwa Chong Institution Boarding School, Tng Jia Hao Barry from Raffles Institution, Khoo Seng Teck of Raffles Junior College and Vivek Sanjay Jain from United World College of South East Asia.