

SIMO Camp 2005

by
Gary Yeh

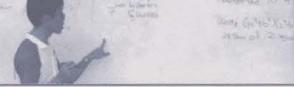
Every year, the Singapore Mathematical Society organises a mathematics camp for secondary school and junior college students during their June holidays. These students are also members of the Senior and National olympiad training teams, which meet regularly on Saturdays at NUS. The goal of the camp is to provide some extra training, and to give the participants a chance to socialise with one another and make friends. This year, the camp was held from 10th to 15th of June, over 6 days and 5 nights. Being a former team member, I decided to help out.

The schedule for each day was roughly as follows:

The campers would wake up for breakfast at around 8 am. Training began at 8:30 am, lasting all the way until 5 pm, so after breakfast, they would take a morning stroll from Raffles Hall (where they were staying) to block S9 in the Science faculty (where their training was held). The Senior team were lectured on various topics such as Number Theory, Inequalities, Geometry, Combinatorics and Graph Theory by the NUS lecturers. The national team focused more on olympiad problem-solving, in preparation for the International Mathematical Olympiad (IMO), which will be held this year in early July in Mexico. There was plenty of food, with lunch, and morning and afternoon tea provided during the training. This year, the food was served buffet-style and catered outside the training room, so the campers did not have to walk far for lunch, and could enjoy the food in air-conditioned comfort. Dinner was also served afterwards, back at Raffles Hall. Each camper was also given a free shirt, with a logo representing this year's D5S camp on it.

Evenings were free and easy. Many campers would play cards or sports, or just relax until dinnertime.





Usually, the highlight of the day was a special programme every night after dinner. For the first evening, we had an ice-breaker session where the campers, especially the new members, got to meet and become familiarised with each other. We played a namememorization game. To put a mathematical twist to things, campers had to, in turn, recite the names of the campers that had gone before them, and also announce a prime number between 1 and 160 which had not been previously announced. There was also a game where each camper was given a list with statements such as 'I know the names of 11 Liverpool mid-fielders', and 'I was born in the same month as you'. They had to find someone else who satisfied that criteria, and get him or her to sign next to it. The last few campers, unable to get enough signatures, had to perform forfeits for the rest of us.

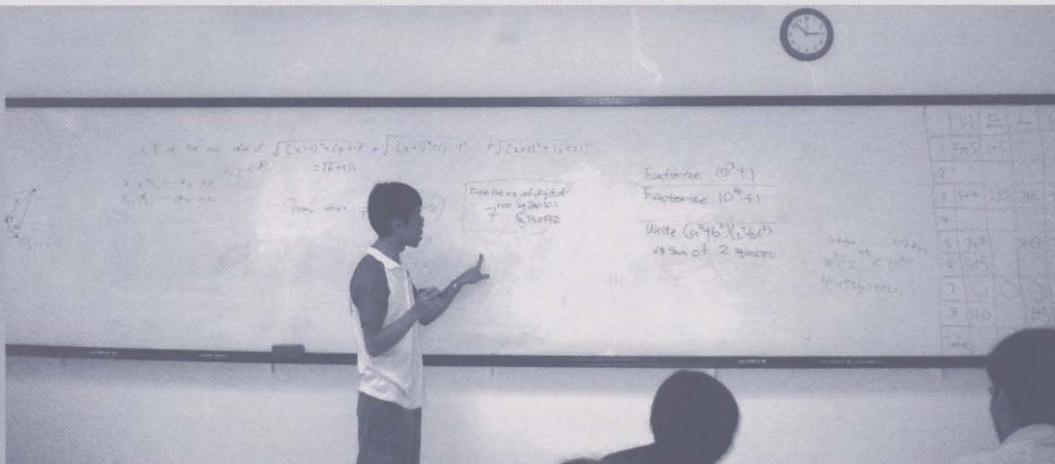
After the ice-breakers, some of us (the author included) stayed behind for a game of Mao. What is Mao? Unfortunately, no one can be told what Mao is. You have to play it for yourself. One brave young camper, new to the game, stoically accepted penalty after penalty as we pounced on him every time he broke a rule. I think he caught on pretty quick though.

We had something called 'Vertical Training' on the second day. Here, the senior team was divided into 6 groups, each of which had to solve a set of problems in the morning. The roles were reversed for the national team. They had to play mentors to the younger members and guide them towards the ideas behind the solutions, as well as teach them how to write up the solutions properly. The afternoon was a period of coordination, where each team would send a (different) representative for each problem that they solved to meet with the national team members, who were the ones marking the questions. As expected, there were many errors, and poorly written parts, so the representatives' job was to present and defend their solutions verbally. Scores were given for each question based on both the written solution and the verbal coordination. I had a lot of fun tormenting the budding mathematicians with simple, yet fiendishly difficult questions about their proofs, such as why two curves had to intersect, or why an order 6 polynomial had only 2 solutions. I think this was a good change of perspective for both the senior and national team. The senior team had a chance to be involved in arguing for marks, which is something that takes place in many international olympiads, including the IMO. As for the national team, they got a taste of how difficult marking proofs can be, and a better appreciation of why a proof might seem perfectly fine to them when they write it, but incomprehensible to an outsider.

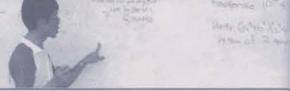
The second night was similar, just that it was the X-men being introduced this time. '4X-men' is the term we use to describe former SIMO training team members, who have graduated from JC, and hence are no longer eligible to participate in competitions such as the IMO (Ex-men). I'm proud to have this title as well. And yes, we like to

think that we do have mutant powers. It's part of the camp tradition that each year, many X-men who have had fond memories of their olympiad training years would come back and share their knowledge, experiences and generally increased wisdom with the young campers on this night. It is always incredible to see the diversity of X-men present. There were '4 young' X-men who were still serving NS, having spared their precious weekend to come down to meet everyone. There were university students such as myself, many of whom were back from abroad for the holidays. And on the other end, there were older X-men who'd finished university, some having completed their Masters and Ph.Ds. And while there was a small 'generation gap' between us and the campers, we hope that we were able to tell them a bit about university, studying further mathematics there, and basically life in general.

For the next two night programmes, Daniel Hue, a fellow X-man and I gave each gave a talk to the campers, on different mathematical topics. On the third night, Daniel talked about 'Computer visualizations of Kleinian groups of Möbius transformations'. Basically, Möbius transformations are a certain kind of transformation of the plane back onto itself. What Daniel tried to show us was how we can picture this transformation of the entire plane, by looking at smaller portions of the plane, and where those portions are brought to by the transformation. While most of us didn't manage to figure out what that title meant exactly, we did get to see the computer visualizations themselves, which were pretty fractal-like pictures. Right after that, we also saw a slide show of pictures taken during the recent South East Asian Mathematical Olympiad (SEAMO), which some of the national team members took part in. This turned out to be a fun night.



The next night was my turn. I talked about a 2-player game called Nim. It's a fairly simple game, played with piles of objects. Both players take turns removing some (possibly all, but at least 1) of the objects from one pile. The player who removes the last object wins. It turns out that Nim has a fairly simply winning strategy, and this strategy can be adapted for many other games as well. In spite of the mistakes I made on the whiteboard, on the whole, I think the campers understood the main ideas.



The third day was also the start of 2 days of IMO-style problem-solving for the national team. Each day, they had 3 questions to solve within 4 and a half hours in the morning.

The questions for both days were:

1. An *alternating sequence* is a subsequence of $(1, 2, \dots, n)$ such that the first term is an odd number, and the terms alternate in parity. How many alternating sequences are there?
2. A series $\{a_i\}$ is such that a_1, a_2 are positive and distinct, and we have $a_n = |a_{n+1} - a_{n+2}|$, for all integers $n \geq 3$. Show that this series is unbounded (i.e. does not have a finite upper bound).
3. Let O be the circumcenter of an acute-angled triangle ABC with $\angle ABC < \angle ACB$. The line AO meets the side BC at D . The circumcenters of the triangles ABD and ACD are E and F respectively. Extend the sides BA and CA beyond A to points G, H respectively such that $AG = AC$ and $AH = AB$. Prove that the quadrilateral $EFGH$ is a rectangle if and only if $\angle ACB - \angle ABC = 60^\circ$.
4. Find all integers $n > 1$ such that $\phi(n)$ divides n . ($\phi(n)$ denotes the number of positive integers at most n that are coprime to n)
5. Circles C_1 and C_2 with centers O_1 and O_2 respectively, meet at the points A and B . Lines O_1B and O_2B intersect C_2 and C_1 at F and E respectively. The line parallel to EF through B meets C_1 and C_2 at M and N . Given that B lies between M and N , prove that $MN = AE + AF$.
6. Prove that in a $nk^n \times nk^n$ square grid, where each cell is coloured with one of k colours, there must exist a $n \times n$ submatrix consisting of cells of 1 colour only. (The rows and columns need not be adjacent)

The afternoon of the fourth day was spent in coordination between the national team members, and the X-men (including me) who marked the questions. Arguments for marks could get quite heated sometimes, but that only goes to show how damaging gaps and ambiguity can be in a proof.

A formal dinner was held on the fifth (and last) night of the camp. Awards for competitions that took place earlier in the year were presented. Of course, the food was good as well. The last night of the camp is always unique. Almost everyone would stay up past the curfew, or even not sleep at all, playing games. The national team was up late as well, but for different reasons. The morning of the last day of the camp (Day 6) is an event called the Oral Quiz, which is arguably the highlight of each camp. It's a buzzer-round style competition traditionally hosted by the national team for the younger members. And it's also traditional (or perhaps more accurately, a consequence of human nature) that the questions for the Oral Quiz are set last minute, on the night before.

The night passed, and the final day began... with heavy rains! This was particularly inconvenient since we had all checked out of our rooms, and had all our luggage with us, making it inconvenient to move around. The campers were stuck at Raffles Hall while the rain didn't show any signs of stopping, until a plan was decided. One of the teachers would ferry our bags to the training room. Two friends and I brought the campers to the internal shuttle bus-stop, under the shelter of very few umbrellas (it is surprising how a large group of supposedly intelligent mathematicians can have so little foresight as to bring so few umbrellas). I think that made this the most memorable Oral Quiz for me.

The campers eventually all reached the training room safely, some wetter than others perhaps, and the Oral Quiz began. The campers were divided into 4 teams, and in place of electronic buzzers, each team had to choose a word or phrase to act as their buzzer. So for instance, one team had to shout 'Duck' before they could answer, while another had to shout 'Ouch!'. While the theme of the Oral Quiz is mathematics, there were plenty of other, weird questions floating around as well. Here are some samples of their flavour:

A part 1) On an island, there are 1000 inhabitants. Some of the inhabitants are one-legged, and they wear one shoe. Half of the remaining inhabitants are two-legged, and they wear two shoes. The rest go barefoot. How many shoes are worn?

A part 2) On an island, there are 1000 inhabitants. They all wear perfectly good shoes. Some of the inhabitants are one-legged, and they wear one shoe. Half of the remaining inhabitants are two-legged, and they wear two shoes. The rest go barefoot. How many shoes are worn?

One interesting question that appeared was:

B) How many SIMO members are going to the IMO this year?

(keeping in mind that every participating country sends 6 contestants each year)

The last question of the Oral Quiz completely surprised everyone:

C) ABC is an equilateral triangle. Let A_1, B_1, C_1 be points on BC, CA and AB respectively. Define points A_2, B_2, C_2 such that we have $A_2B_2 \parallel A_1B_1$, $B_2C_2 \parallel B_1C_1$, $C_2A_2 \parallel C_1A_1$, and C lies on A_2B_2 , B lies on A_2C_2 and A lies on B_2C_2 . Let A_3, B_3, C_3 be the respective feet of the altitudes from A_2, B_2, C_2 to sides B_2C_2, C_2A_2, A_2B_2 . Let R_1, R_2, R_3 be the respective incenters of triangles ACB_2, ABC_2, BCA_2 . Let O is the circumcenter of $\triangle ABC$, and P_0 the reflection of O about BC . Let $R_1R_2D_2D_1$ be a rectangle such that $R_1D_1 = OP_0$. Assume the minor angles D_2R_2O and D_2B_1O sum to 180° , and minor angles $P_0C_3B_1$ and P_0OB_1 sum to 180° . Given that $R_2P_0C_3D_2$ form a cyclic quadrilateral, find 3 lines containing 6 distinct alphabets such that the 3 lines are concurrent.

Some hints that were given out for this monster:

“Episode III is out”

“Droid 1, Droid 2, Droid 3, Droid 4, ...”

“Actually, Droid 3 isn't really a Droid”

And with that came the conclusion to this year's SIMO Camp. But beyond the camp, there's more I'd like to talk about as well. The SIMO members, both current and former (X-men), form a very tight-knit community. The camp is simply one aspect of this; most of the camp participants would be going through senior and/or national team training for a few years. Looking back at all this, I've realised that it is a very wonderful to be able to share something you enjoy (I'm pretty sure almost everyone in SIMO loves mathematics) with like-minded people. Usually, it's the same friends you make in the senior team that will also be with you in the national team, or even at the IMO. In fact, my former IMO teammates and I have been helping out with the national team training since the start of this year, and they'll be going with the IMO team as observers. I think we've enjoyed conducting the training as much as the national team has enjoyed being trained. If you do get a chance to join the senior or national teams, I hope you won't turn it down.

While this may seem cliché, there are many people whom I'd like to thank.

Chen Zhihao, for the photographs. Calvin Lin, for helping me recall many of the events of the camp. Julius Poh and Charmaine Sia, for courage under fire (or rather, water!). And the other campers and X-men. It has been my great pleasure to be part of all this again.

Some Solutions

A1) 1000

A2) None. All were perfectly good.

B) Of course, the 6 IMO team members this year: Kenneth Tay, Charmaine Sia, Daniel Chen, Zhao Yan, Soh Yong Sheng and Wu Jia Wei. Also, the deputy team leader this year, Mr Koh Yi-Huak is a former SIMO member (having represented Singapore in the IMO in 1992). The three observers going, Calvin Lin, Julius Poh and Tan Kiat Chuan, are likewise former representatives. And on top of all that, one of last year's Singapore team members, Andre Kueh, is now studying in the UK, and will be representing UK this year! So that gives a total of 11 SIMO team members.

C) R_2D_2, C_3P_0 and OB_1 . And yes, the lines do concur. The question was created by Zhao Yan.

Editor's note:

Our 2005 IMO team has done very well. They brought home 4 silver medals and 2 bronze medals! Unofficially, Singapore was ranked 14th, amongst 91 countries.