

## Introduction

The Distinguished Visitor Programme was launched by the Singapore Mathematical Society in 1998. Through the visit of a distinguished mathematician/mathematics educator and his/her interaction with both mathematicians/mathematics educators at the universities as well as teachers and pupils at the schools here, the aim of the Programme is to expose as large and diverse an audience as possible to the excitement and relevance of mathematics, thereby enhance the awareness of mathematics in our society.

In 2001, the Society was privileged to have as its Distinguished Visitor Professor Wilfried Schmid. Born in Hamburg, Germany, and educated at Princeton University and the University of California at Berkeley, USA, Wilfried Schmid attained full professorship at Columbia University at the age of 27. In the same year, Professor Schmid was invited to speak at the International Congress of Mathematicians (the first of three consecutive occasions in the quadrennial event), one of the highest honours bestowed on a mathematician by his peers. Since 1978, Wilfried Schmid has been Professor at Harvard University, occupying the Dwight Parker Robinson Chair since 1985.

Besides mathematical research, Professor Schmid is also a leader in mathematics education. He currently serves as mathematics advisor to the Massachusetts Department of Education, and also sits on the Steering Committee for Mathematics of the US National Assessment of Educational Progress (NAEP, which produces the so-called "Nation's Report Card"), and the International Program Committee of the 10th International Congress of Mathematical Education (Copenhagen, 2004). Professor Schmid's article "New Battles in Math Wars" on mathematics education reforms, which can be obtained at the website <http://www.math.harvard.edu/~schmid/articles/wars.html>, appeared in the Harvard Crimson in May 2000.

During his visit to Singapore, Professor Schmid gave the SMS Distinguished Visitor Lecture entitled "A Critical Look at Mathematics Education in the United States", a dialogue session on "Some Issues on Mathematics Education", a colloquium lecture on "Automorphic Distributions" at NUS, as well as engaged in exchange of views and ideas on mathematics education with teachers, students,

Professor Wilfried Schmid,  
Harvard University

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officials and researchers in the educational sector, and the general public. A summary of Professor Schmid's lecture on "A Critical Look at Mathematics Education in the United States" is recorded below.

## US School System

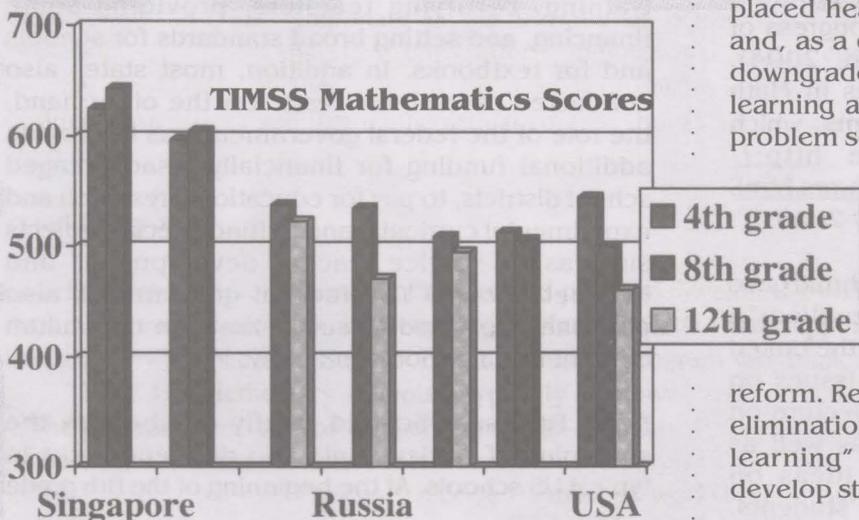
To familiarize his audience with the US school system, Professor Schmid began his lecture by looking at the organization of public schools in the United States. Although details might vary by state, he pointed out that typical public schools were administered and financed mostly by the communities. Consequently, per-student spending depended on the wealth of the community, resulting in highly non-uniform public schools. He also outlined the different roles in education played by the states and the federal government. The role of the states included setting standards for teacher training, certifying teachers, providing some financing, and setting broad standards for schools and for textbooks. In addition, most states also conducted assessment tests. On the other hand, the role of the federal government was to provide additional funding for financially disadvantaged school districts, to pay for educational research and experimental curricula, and to fund special projects such as "in-service teacher development" and internet access. The federal government also published non-binding suggestions on curriculum content and textbooks recently.

Next, Professor Schmid briefly touched on the streaming of students into two different tracks in typical US schools. At the beginning of the 8th grade

(equivalent to Secondary 2 in Singapore), students were streamed into two parallel tracks: a single-discipline track where students were taught courses in Algebra, Geometry, Pre-calculus and Calculus; and an 'integrated' track where mathematics was usually taught at a lower level. He told the audience that mathematics was typically taught by specialized mathematics teachers only in 5th grade or higher. With regard to the training of mathematics teachers, he said that mathematics teachers were trained in Schools of Education, and future teachers, including future high school mathematics teachers, were usually taught mathematics by mathematics educators (not mathematicians), who placed much more emphasis on pedagogy rather than content knowledge.

## Third International Mathematics and Science Study

Professor Schmid then turned his attention to the Third International Mathematics and Science Study (TIMSS) 1995. The TIMSS was an elaborate international comparison of mathematics and science education in different countries, including Singapore, Japan, France, Russia, Germany, England and the United States. The study collected a large amount of data and employed an unusually careful methodology in comparing student performance, teacher preparation, textbooks and teaching styles. He showed the audience a bar chart that compared the mathematics scores of students in 4th, 8th and 12th grades taking part in the TIMSS from the above-mentioned countries. He also included a few sample questions (appended at the end of this article) from 8th and 12th grades and the percentages of correct responses to each sample question from students from different



countries. Professor Schmid noted that the performance of US students declined drastically at later grades.

Several conclusions were drawn from the TIMSS about US student performance and US textbooks. First of all, the US students did relatively well on one-step problems, and on "data analysis" problems which were emphasized in the United States. However, they did badly on multi-step problems and problems that required conceptual thinking. As for the US textbooks, they were less cohesive, with frequent breaks between subtopics. Furthermore, the number of pages in a typical US textbook was also much larger; and the number of topics covered in any one year was greater, with the topics remaining in the curriculum (from year to year) much longer than in other countries. Professor Schmid said that the problems documented by TIMSS were at least partially understood already in the 80s, and he cited as examples two reports: a 1983 report "A Nation at Risk" by the US government commission, and a 1989 report "Everybody Counts", which contained recommendations by the National Research Council.

## National Council of Teachers of Mathematics

Professor Schmid also mentioned the emergence of a new player in the eighties - the National Council of Teachers of Mathematics (NCTM), which was a professional organization of mathematics teachers. The NCTM issued curriculum guidelines in 1989 that aimed to reform the teaching of mathematics in the United States. It was an elaborate document written by a large committee of mathematics educators and teachers, and was promoted by supporters as de facto national mathematics curriculum guidelines. The NCTM 1989 guidelines placed heavy emphasis on the use of calculators, and, as a consequence, computational skills were downgraded. The guidelines also promoted group learning and discovery learning, and emphasized problem solving as a key to mathematical learning. "Data analysis" and "statistics" became important topics, and proofs were almost completely eliminated.

Following the issue of the NCTM 1989 guidelines, there was a heated debate between reformers and skeptics of the reform. Reformers demanded the reduction or even elimination of direct instruction in favour of "group learning" and "discovery learning" that would develop students' mathematical thinking. They also

demanded that calculators be used at all times and less emphasis be placed on paper-and-pencil computations. In contrast, skeptics of the reform thought that computational skills and memorization were still very important in the learning of mathematics, and that calculators should be used sparingly until after computational competence was attained. They believed that there should be a mix of instructional techniques, including direct instruction, and that mathematical content should not be neglected in the race to reform.

## A Reformed Program

Professor Schmid provided an example of a reformed program developed by TERC, a non-profit educational think tank in Cambridge, Massachusetts, and supported by a \$7,000,000 grant by the National Science Foundation. The program was implemented by roughly 3% of US elementary schools - typically in liberal and affluent school districts. The program consisted of teacher's manuals - roughly 10 per year, of up to 120 pages each, and training sessions for teachers conducted by TERC. There were no textbooks for students; instead, students received copies of worksheets provided by TERC. The authors of the program claimed adherence to the 1989 NCTM guidelines, and they opposed the teaching of standard algorithms. Professor Schmid criticized the program as being highly scripted, in the sense that teachers were told exactly how to conduct lessons.

Professor Schmid then showed the audience quotes from the TERC manual to give them a sense of the TERC program. According to the TERC manual, in an old-style class, students worked alone, focused on getting the right answer and recorded it by writing down only numbers, used a single prescribed procedure for each type of problem, used only pencils and papers, chalk and chalkboard as tools. The TERC program demanded a new-style class where students worked in a variety of groupings, communicated about mathematics orally, in writing and by using pictures, diagrams and models, used more than one strategy to solve problems, used cubes, blocks, measuring tools, calculators and a large variety of other materials. The TERC manual also spelled out explicitly the new role of a teacher: to observe and listen carefully to students; to try to understand how students were thinking; to help students articulate their thinking, both orally and in writing; to establish a classroom atmosphere in which high value was placed on thinking hard about a problem; to ask questions that pushed students' mathematical thinking further; to facilitate class discussion about important mathematical ideas. Professor Schmid made a comment that the

program emphasized multiple strategies in solving problems to the extent that students did not learn any primary method in the end. He also gave a few examples from TERC worksheets for 3rd and 5th grade students to illustrate his point.

Professor Schmid was highly critical of the TERC program, claiming that it had too much play and too little substance, that students were kept dependent on mental crutches (like fingers, clock faces, blocks etc.), and that the practice problems were far too few and far too easy. He thought that the intellectual level was too demeaning to bright students, while students with poor verbal skills were disadvantaged.

## Use of Singapore Mathematics Textbooks in US

Professor Schmid then touched on the use of Singapore mathematics textbooks in the United States. He told the audience that the Singapore mathematics textbooks were widely used by home schoolers, as well as in a significant number of public and private schools on an experimental basis. The experiments so far had been generally encouraging. The Singapore textbooks also served as a useful reference point for debate about mathematics education in the United States. Professor Schmid pointed out some obstacles to the widespread adoption of Singapore textbooks in the US: the extensive training required of teachers; the need to start from 1st grade and maintain a long-term commitment; not enough coverage of certain topics - for example, data analysis - that were emphasized in the US.

## Conclusion

To conclude his talk, Professor Schmid offered his views on ingredients of a good mathematics education: well-trained teachers; balance between computational practice, problem solving and conceptual understanding; sensible balance between direct instruction and discovery learning; good textbooks; addressing the needs of students with various degrees of mathematical talent; putting some pressure on students but not too much.

Editor's Note: Interested readers may like to view Professor Schmid's lecture which can be accessed at <http://sms.math.nus.edu.sg/visitors/visitors2001.html>.

## TIMSS sample questions, 8th grade

Solve linear equation for x: if  $3(x+5) = 30$ , then

- a)  $x = 2$     b)  $x = 5$     c)  $x = 10$     d)  $x = 95$

Correct responses: Singapore 96%; Japan 90%; France 82%; Russia 88%; Germany 79%; England 61%; USA 73%

If  $m$  represents a positive number, which of these is equivalent to  $m + m + m + m$  ?

- a)  $m + 4$     b)  $4m$     c)  $m^4$     d)  $4(m+1)$

Correct responses: Singapore 82%; Japan 75%; France 65%; Russia 75%; Germany 57%; England 42%; USA 46%

## TIMSS sample questions, 12th grade

If there are 300 calories in 100 grams of a certain food, how many calories are there in a 30 gram portion of that food?

- a) 90    b) 100    c) 900    d) 1000    e) 9000

Correct responses: Netherlands 84%; France 80%; Russia 71%; Germany 74%; USA 68%

Brighto soap powder is packed in cube-shaped cartons. A carton measures 10 cm on each side. The company decides to increase the length of each side by 10%. How much does the volume increase?

- a) 10 cm<sup>3</sup>    b) 21 cm<sup>3</sup>    c) 100 cm<sup>3</sup>    d) 331 cm<sup>3</sup>

Correct responses: Netherlands 50%; France 31%; Russia 30%; Germany 25%; USA 17%

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