Mathematics : The New Golden Age

• *By* ***Keith Devlin, Columbia University Press, 1999, xi + 320 pp.***

This is a revised and updated version of the book with the same title published by Penguin in 1988. During the decade since then, many new developments in mathematics and physics have taken place and some important mathematical problems that have defeated generations of the best minds are either solved or have conceded significant ground to the relentless and combined efforts of many mathematicians all over the world. The mathematical enterprise, like its scientific counterpart, has become a collective activity and it is estimated that 90 per cent of all mathematicians who ever lived are still living today. Little wonder that mathematics is entering a new golden age, to which this book bears testimony.

Devlin is one of the few mathematicians who have not only contributed to advancing the frontiers of mathematics but also to popularizing and raising the awareness of mathematics among the general public through books, newspapers, radio and television. He has brought the latest mathematical breakthroughs to the doorsteps of the layman and succeeded in conveying the accompanying intellectual excitement that is akin to the excitement following the trails of the conquest of inaccessible mountain summits or of the exploration of unchartered territory.

By the nature of the subject, it is practically impossible for the author to communicate intelligibly to the layman without a substantial amount of concepts and terminology. In this book he has not flinched from doing so. There are enough details to make the mathematical specialist feel at home. The layman will undoubtedly feel daunted by the unfamiliar and abstract ideas while the budding mathematics student will feel intoxicated by an otherworldly sensation of a different realm.

The book begins with prime numbers and ends with algorithms. In between, it talks about logic, set theory, geometry, topology, physics and linear programming. It tells you of the long and arduous path taken by problem solvers and trail blazers, like the story of how the famous 350-year-old conjecture called "Fermat's Last Theorem" was finally settled or how the less sensational but equally tantalizing story of how another problem in number theory called the "class number problem" was laid to rest after 183 years.

Mathematics thrives in the convulsions created in the wake of attempts to solve, successfully or not, those problems identified as landmark problems. Without such problems, an area of mathematics will shrivel and become barren. This book identifies some of these problems that have been solved or for which significant progress in knowledge has been made in the twentieth century in number theory, set theory, geometry, algebra, logic, graph theory, analysis, topology and combinatorics. Each problem often has a long and tantalizing history of spoofs, human folly and relentless struggle before the problem was subdued. Its history is told in a captivating manner, and the aficionado is directed to more detailed sources at the end of each chapter. The less mathematically minded reader could skip the technical details and still get some glimpse of the thorny paths taken by the problem solvers.

Among the landmark problems mentioned are the primality testing problem, the class number problem and Fermat's Last Theorem in number theory, the continuum hypothesis in set theory, the solvability of the general polynomial equation by radicals, the classification of finite groups and Hilbert's Tenth Problem in algebra, the four-colour problem in graph theory, the Riemann hypothesis and the Bieberbach conjecture in complex analysis, the classification of knots and the Poincare conjecture in topology and some NP-complete problems in computability theory. If you are curious about the meaning of these mathematical terms, you will find your curiosity more than whetted when you delve into the book. Even the mathematics specialist will be unlikely to be familiar with more than 50 percent of these terms. For him or her too, this book will expand his or her mental horizons.

Most laymen believe that everything is clear-cut in mathematics, that something is either right or wrong. Yet the history of mathematics has shown that when it comes to deciding whether the solution of a difficult problem is correct or not, things may not be so clear-cut. The most striking example is the case of the young French genius Evariste Galois (1811 - 1831). His ground-breaking solution of an old algebraic problem was repeatedly rejected or ignored by the ruling high priests of the French mathematics circle because none of them could understand the novelty and depth of his ideas. His ideas were almost lost to posterity and were painstakingly resurrected 12 years after his tragic and untimely death.

Devlin mentions another less dramatic example of non-recognition and neglect by the mathematical community of Heegner's solution in the class number problem in 1952. Heegner was a retired Swiss scientist when he did mathematics as a hobby. Nobody believed his solution because they could not understand it. He was vindicated only 15 years later by two prominent number theorists (Harold Stark and Alan Baker) who looked at his solution after they had themselves independently solved the same problem by different methods. A more recent case is that of the American mathematician Louis de Branges whose solution of the Bieberbach conjecture in 1984 was rejected by his own countrymen. He was, however, fortunate enough to have presented his work not long afterwards and gotten it scrutinized and found to be correct by Soviet mathematicians during a short visit to Leningrad.

The book draws out some of the drama and intrigue behind the solution of famous problems. Even with all the lofty aims and ideals attributed to the pursuit of mathematical truths, mathematical activity is still very much a human activity. After all, mathematicians are human beings too.

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