The 37th International Mathematical Olympiad

*Reported by **Qu Ruibin***

The 37th International Mathematical Olympiad (IMO) was held in Bombay, India from 5 to 17 July 1996. 75 teams of 426 contestants representing 75 countries and regions participated in the competition. The Singapore National Team to the 37th IMO consisted of the following.

Team Leader:

Dr. Qu Ruibin
(National University of Singapore)

Deputy Leader:

Mr. Gui Eng Hock
(Serangoon Junior College)

Contestants:

Lim Yee Fun
(Raffles Institution)

Low Tzer Hung
(Raffles Junior College)

Pang Chin How, Jeffrey
(Hwa Chong Junior College)

Senkodan Thevendran
(Raffles Junior College)

Tan Kwang Pang, Daniel
(Raffles Junior College)

Yeo Keng Hee
(Raffles Institution)

The Singapore Team to the 37th IMO with the Team Leader

From July 6 to 9, six problems were selected for the 37th IMO competition from 30 shortlisted problems by the 37th IMO Jury made up of all the 75 team leaders. These problems were chosen by the Jury very democratically (through voting). When the English version of the problems was finalized, it was translated by the leaders into 44 other languages suitable for their contestants. All these versions were approved by the Jury to certify their appropriateness of translations in the afternoon of July 9, 1996.

The first paper of the 37th IMO competition started at 09:30 hour and ended at 14:00 hour on July 10, and the second paper was tested at the same time the next day. All the answer sheets were given to the team leaders at 21:00 hour on July 11. The leaders and deputy leaders had to start immediately reading and marking the scripts of their students because coordination of the marking schemes was scheduled the next day at 9:00 hour. As a result, most of the team leaders and deputy leaders had little sleep that night.

Within 48 hours, all the answer scripts were marked and coordinated with the help of more than 44 coordinators from various parts of the host country. The final scores of all the participants were finalized and approved by the Jury just before 22:00 hour of July 14. The cut off points for different medals were also determined: 28 to 42 marks for gold, 20 to 27 marks for silver and 13 to 19 marks for bronze.

At the closing ceremony in the afternoon of July 16, 35 gold medals, 66 silver medals and 99 bronze medals were awarded to the winners of the 37th IMO. Our national team received four medals, one gold (Thevendran) and three bronze (Jeffrey, Tzer Hung and Yee Fun). With total marks of 86, our national team was unofficially ranked 25th among the 75 participating teams.

The 37th IMO is significant to Singapore because it is the first time that our national team has been awarded a gold medal since we started participating in the IMO competition in 1989.

The 37th IMO was thought to have been the most difficult one to date, judging from the score statistics of the participants. For example, for Problem 5, 308 of them got an egg and only 6 of them were awarded a full mark 7. The national team of China, the best one for the 36th IMO in Canada, grabbed nothing from this problem. Only 18 of the 75 participating teams went home with at least one gold medal, and only one of the 426 contestants, a Romanian, obtained a full mark of 42.

The Score Statistics of Contestants

Problem \ Scores	0	1	2	3	4	5	6	7	Average
1	65	30	98	26	111	17	12	63	3.19
2	173	124	10	5	11	6	2	88	2.05
3	48	176	73	22	14	19	19	52	2.40
4	177	109	11	17	11	7	6	26	1.13
5	308	73	18	7	4	4	0	6	0.50
6	195	80	15	15	3	6	10	99	2.25

The 37th IMO Problems

First Day

1. Let $ABCD$ be a rectangular board with $|AB| = 20$, $|BC| = 12$. The board is divided into 20 x 12 unit squares. Let r be a given positive integer. A coin can be moved from one square to another if and only if the distance between the centres of the two squares is $\sqrt{r}$. The task is to find a sequence of moves taking the coin from the square which has A as a vertex to the square which has B as a vertex.

   (a) Show that the task cannot be done if r is divisible by 2 or 3.

   (b) Prove that the task can be done if $r = 73$.

   (c) Can the task be done when $r = 97$?

2. Let P be a point inside triangle ABC such that $\angle APB - \angle ACB = \angle APC - \angle ABC$. Let D, E be the incentres of triangles APB, APC respectively. Show that AP, BD and CE meet at a point.

3. Let $S = \{0, 1, 2, 3, \ldots\}$ be the set of non-negative integers. Find all functions f defined on S and taking their values in S, such that

   $$f(m + f(n)) = f(f(m)) + f(n) \text{ for all } m, n \text{ in } S.$$

Second Day

4. The positive integers a and b are such that the numbers $15a + 16b$ and $16a - 15b$ are both squares of positive integers. Find the least possible value that can be taken by the minimum of these two squares.

5. Let $ABCDEF$ be a convex hexagon such that AB is parallel to ED, BC is parallel to FE and CD is parallel to AF. Let R_A, R_C, R_E denote the circumradii of triangles FAB, BCD, DEF respectively, and p denote the perimeter of the hexagon. Prove that

   $$R_A + R_C + R_E \geq \frac{p}{2}.$$

6. Let n, p, q be positive integers with $n > p + q$. Let $X_0, X_1, \ldots, X_n$ be integers satisfying the following conditions:

   (a) $X_0 = X_n = 0$;

   (b) for each integer i with $1 \leq i \leq n$, either $X_i - X_{(i-1)} = p$ or $X_i - X_{(i-1)} = -q$.

   Show that there exists a pair (i, j) of indices with $i < j$ and $(i, j) \neq (0, n)$, such that $X_i = X_j$.