Problems Corner

CONTEST

Prizes in the form of book vouchers will be awarded to the first received best solution submitted by secondary school or junior college students in Singapore for each of these problems.

To qualify, secondary school or junior college students must include their full name, home address, telephone number, the name of their school and the class they are in, together with their solutions.

Solutions should be typed and sent to: The Editor, Mathematical Medley, c/o Department of Mathematics, National University of Singapore, Kent Ridge, Singapore 119260; and should arrive before 30 June 1996.

The Editor's decision will be final and no correspondence will be entertained.

Problem 1

For each positive integer n, let $\overline{n}$ denote the product of all positive integers less than or equal to n and of the same parity as n. For example $\overline{5} = 5 \bullet 3 \bullet 1 = 15$ and $\overline{6} = 6 \bullet 4 \bullet 2 = 48$. Prove that $\overline{1996} - \overline{1995}$ is divisible by 1997.

Prize

One \$50 book voucher

Problem 2

There are $2n$ cards numbered by $1,1,2,2,3,3,4,4,\ldots,n,n$. How many ways are there to group these cards into n pairs such that the two numbers in each pair are either equal or differ by 1?

Prize

One \$100 book voucher

Solutions

to the problems in Volume 22, No. 2 September 1995

Solution to Problem 3

by Jeremy Wong, St. Gabriel's Secondary School, Class 4A.

Let the time when the robots meet be t seconds.

Then, the distance travelled by the robots would be $21t$, $19t$ and $(53/3)t$. As all three robots start together and are travelling around the same track, hence, when they are together again, they will be at the same distance away from the starting point. The only difference is that the faster ones will have travelled a number of rounds more than the slower ones.

Therefore, if f denotes the distance travelled by the faster robot and s the distance travelled by the slower robot, then

$$\frac{f-s}{\textit{circumference}} = \textit{number of more rounds travelled by the faster robot}$$

Then, $(21t - 19t)/300 = a$ (an integer)

and $(21t - (53/3)t)/300 = b$ (also an integer).

Hence, $t = 150a$

and $t = 90b$.

$\therefore 150a = 90b$

$a/b = 3/5.$

Therefore, the next time the robots meet will be when $a = 3$ and $b = 5$, which implies $t = 450$. When $t = 450$, number of rounds travelled by the fastest robot $= 21(450)/300 = 31.5$.

As the decimal is 0.5, hence when they are together again, **they are half way round the track, i.e. 150m from the starting point.**

Solved also by Joyce Chan, Nanyang Girl's High School, Class 3/6. Chong Chin Yuan, River Valley High School, Class 3D. Huah Cheng Jiann, Dunman High School, Class 3A. Loke Wing Fai, St. Andrew's Secondary School, Class 4 SB. Ma Yun, Nanyang Girl's High School, Class 4/4. Yau Woon Foong, Victoria School, Class 4I.

Editor's Note: Ma Yun and Jeremy Wong shared equally the prize of $50 book voucher.

Solution to Problem 4

by Ma Yun, Nanyang Girl's High School, Class 4/4.

$$XY/AB = CX/CA = CE/CA + EX/CA$$

$$CE/CA = (CA - AE)/CA = 1 - AE/CA.$$

Because AEG and ACB are similar triangles, so $AE/CA = EG/CB = h$.

$$\therefore CE/CA = 1 - h.$$

Because XEP and ACB are similar triangles, so

$$EX/CA = XP/AB = AF/AB = (AB - BF)/AB = 1 - BF/AB.$$

Because FDB and ACB are similar triangles, so $BF/AB = FD/AC = k$.

$$\therefore EX/CA = 1 - k.$$
$$\therefore XY/AB = (1-h) + (1-k) = 2 - h - k.$$

The ratio $XY{:}AB$ is $(2 - h - k)$.

Solved also by Herman Chow, Anglo-Chinese School, Class 4A3. Huah Cheng Jiann, Dunman High School, Class 3A. Xu Jin, Nanyang Girl's High School, Class 3/4. Jeremy Wong, St. Gabriel's Secondary School, Class 4A. One incorrect solution was received.

Editor's note: Herman Chow, Ma Yun, Xu Jin and Jeremy Wong shared equally the prize of $100 book voucher.