THE 31st INTERNATIONAL MATHEMATICAL OLYMPIAD

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The 3st International Mathematical Olympiad (IMO) was held in Beijing, China, from 8 to 19 July 1991. A total of 308 students from 54 countries participated in this test of mathematical skill and ingenuity.

The IMO is organised and held in different countries each year. Last year, it was held in Brunswick (Braunschweig), Germany [see Koh Khee Meng, Tay Tiong Seng, *Singapore's participation in the 30th IMO*, Mathematical Medley **17** (1989), pp 79 - 88)]. Next year, it will be held in Sigtuna, Sweden. The main objectives of the IMO are [from *Regulations of the 31st International Mathematical Olympiad*]:

(1) "the discovering, encouraging and challenging of mathematically gifted school students throughout the world;
(2) "the fostering of friendly international relations between students and teachers;
(3) "the creating of an opportunity for the exchange of information on teaching and practice in all countries."

Participation in IMO is by invitation, and the invited country may send a national team of at most 6 contestants who are students not formally enrolled at a university and who are not older than 20 years of age at the time of the competition.

The Singapore team to the 31st IMO consisted of 6 students: Hsi Hanyin (Hwa Chong JC), Jin Zihuai (Temasek JC), Lin Ziwei (Chinese High School), Lim Li Woon (Raffles JC), Tan Chong Hui (Raffles Institution) and Yu Changkai (Victoria JC). The team was led by Dr Leong Yu Kiang (Team Leader) and Dr Tara R. Nanda (Deputy Leader) from the National University of Singapore. Mr Song Hoe Chye, a teacher from Chinese High School, accompanied the team to observe the proceedings of the IMO. Of the team members, Changkai had taken part in last year's IMO. For the rest it was a completely new experience.

The contest was held in the Beijing Language Institute on 12 and 13 July from 9.00 am to 1.30 pm. On each day of the contest, students were

given $4\frac{1}{2}$ hours to solve 3 problems. The 6 problems of the IMO were selected from a list of problems submitted by the participating countries (the host country did not propose any problem). The organising committee first made a preliminary selection of 28 problems and everyday, from 8 to 11 July, team leaders from participating countries convened meetings to decide the final 6 from this preliminary selection. The discussion was conducted essentially in English with translation into French and occasionally into Russian. It took 6 meetings to agree on the choice and final text of the 6 problems in English, French, Russian and Spanish. There were also translations of these problems into the various native languages of participating countries by the respective team leaders.

During the preparation of the olympiad problems team leaders were physically separated from the rest of the team. Only after the first examination did the deputy leaders move into the same hotel (Fragrant Hill Hotel) where the team leaders were staying.

Because of the diversity of languages used, team leaders had to answer queries raised by their respective contestants on the two days of the contest. This was done strictly through official couriers who had to ply back and forth from examination rooms to the language laboratory where the team leaders were assembled. Replies to contestants' queries had to be approved by the "jury" as the organisers and team leaders were collectively referred to.

The assessment of contestants' scripts was done by the respective team leaders and deputy leaders. The final marks awarded had to be approved by panels of coordinators who were Chinese mathematicians from various parts of China. Each panel comprised two mathematicians and was responsible for one specific problem. So for two full days team and deputy leaders acted as counsels for their students in order to obtain the marks their solutions were supposed to deserve. The Chinese contestants themselves were coordinated by separate panels formed from the team leaders of certain other countries. In the course of the coordination, individual results were entered on posters displayed outside the meeting (coordination) hall. This generated a lot of excitement and expectation. A Japanese television crew was apparently seen to be "shooting" the results. (Japan took part in the IMO this year for the first time.) Much publicity was also given in the Chinese news media to the proceedings of the IMO. There was a feature article in the 20 July English edition of the *China Daily* on some aspects of the Chinese mode of training for the IMO.

Of course, it was not all work and competition. There was plenty of sightseeing in and around the historic city of Beijing - Summer Palace, Palace Museum, Temple of Heaven, Ming Tombs, Great Wall at Badaling, etc. There were 3 banquets, the last being held at the Great Hall of the People, an acrobatic performance (at the opening ceremony), a Beijing opera performance and a symphony orchestra concert (the latter at the end of the medal presentation).

At each IMO, only individual awards are given. Other than gold, silver and bronze medals, honourable mentions are presented to those who do not qualify for the above-mentioned medals but who score full marks for at least one problem. A special prize is awarded to any contestant who submits an outstanding solution. This year, however, no special prizes were awarded. Generally, not more than half the number of contestants are awarded medals, and the numbers of golds, silvers and bronzes awarded are approximately in the ratio of 1:2:3.

With a maximum mark of 7 for each problem, the maximum individual score is 42. For this year's performance, the ranges of individual score qualifying for the medals are:

Gold : 34 - 42; Silver : 23 - 33; Bronze : 16 - 22.

From the data which I collected from the score charts, I found that the numbers of golds, silvers and bronzes awarded are 23 (7.5%), 57 (18.5%) and 75 (24.4%) respectively, thus giving a total of 155 (50.3%) medals awarded. The number of honourable mentions presented is 33 (10.7%). There were 4 perfect individual scores: 2 from China, 1 from USSR and 1 from France. The only girl with a perfect score came from USSR; this was, I believe, her third participation in IMO. I have compiled an unofficial breakdown of individual awards and total team score from my records (see Appendix). I have also computed the mean, variance and standard deviation of the total score obtained by each contestant. This is done on the assumption (or convention) that a score of 0 is entered for a problem not attempted.

Mean of total score of contestant = 17.16

Variance of total score of contestant = 88.40

Standard deviation of total score of contestant = 9.40

Median of total score of contestant = 15.06

Lim Li Woon and Tan Chong Hui were each awarded a bronze medal while Jin Zihuai and Yu Changkai were awarded honourable mentions. Hsi Hanyin and Lin Ziwei both missed the bronze by one mark each. In the unofficial ranking by total team score given in the Appendix, the Singapore team ranks 27th out of 54. The average score obtained by our team members is 15.5.

Since IMO is essentially a test of individual skill, it would be interesting to analyse the relative difficulty of the 6 problems. Below is a frequency table for these problems together with the computed means, variance and standard deviation.

31st IMO Individual Scores (Unofficial)						
Problem	# 1	# 2	# 3	# 4	# 5	# 6
N_0	88	48	25	31	21	115
N_1	54	45	118	75	39	93
N_2	28	37	79	71	43	29
N_3	19	54	42	46	31	31
N_4	26	11	16	10	22	18
N_5	14	9	6	3	26	2
N_6	8	8	6	3	26	3
N_7	71	96	16	69	100	17
Mean	2.88	3.54	2.09	2.96	4.19	1.50
Variance	7.56	7.21	2.84	5.87	6.22	3.49
Stdev	2.75	2.69	1.68	2.42	2.49	1.87

(N_i = Number of contestants with score i, $i = 0, 1, \ldots, 7$)

(Stdev = standard deviation)

If we use the z-statistic

$$z = \frac{|\mu_1 - \mu_2|}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}}$$

we obtain the following table of values of z for each pair of problems. At the level of significance $\alpha = 0.15$, we see that Problems 1 and 4 are of the same level of difficulty and that other pairs of problems are of different level of difficulty. The problems appear to have a wide range of difficulty:

Problem 5 (a combinatorial game problem) being the easiest and Problem 6 (a combinatorial geometrical problem) the hardest.

Computed values of z-statistic						
	# 1	# 2	# 3	# 4	#5	# 6
# 1		3.01	4.30	0.38	6.19	7.29
# 2			8.03	2.81	3.11	10.94
# 3				5.17	12.24	4.12
# 4					6.21	8.38
# 5						15.15

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The Singapore Team: (from left to right) Jin Zihuai, Lim Li Woon, Yu Changkai, Tan Chong Hui, Lin Ziwei, Hsi Hanyin.

Appendix

31st IMO Results (Unofficial)

Total number of candidates = 308
Total number of medals awarded = 155 (50.3%)
Number of gold awarded = 23 (7.5%)
Number of silvers awarded = 57 (18.5%)
Number of bronzes awarded = 75 (24.4%)
Number of honourable mentions awarded = 33 (10.7%)

Team = Team score (Maximum = 42)
G = Gold (34 - 42), **S** = Silver (23 - 33), **B** = Bronze (16 - 22)
HM = Honourable Mention (Full score for one question but ineligible for any medal)

	Country	Team	G	S	B	HM
1.	China	230	5	1	-	-
2.	USSR	193	3	2	1	-
3.	USA	174	2	3	-	-
4.	Romania	171	2	2	2	-
5.	France	168	3	1	-	1
6.	Hungary	162	1	3	2	-
7.	East Germany	158	-	4	2	-
8.	Czechoslovakia	153	-	5	1	-
9.	Bulgaria	152	1	4	1	-
10.	United Kingdom	141	2	-	2	1
11.	Canada	139	-	3	1	2
12.	West Germany	138	-	2	4	-
13.	Italy	131	1	1	4	-
14.	Iran	122	-	4	-	-
15.	Australia	121	-	2	4	-
16.	Austria	121	-	1	4	1
17.	India	116	1	1	2	-
18.	Norway	112	-	3	1	-
19.	North Korea	109	-	1	3	-
20.	Japan	107	-	2	1	-
21.	Poland	106	-	2	1	2
22.	Hong Kong	105	-	-	4	1

	Country	Team	G	S	B	HM
23.	Vietnam	104	-	1	3	-
24.	Brazil	102	1	-	2	-
25.	Yugoslavia	98	-	1	2	1
26.	Israel	95	-	1	3	-
27.	Singapore	93	-	-	2	2
28.	Sweden	91	-	1	2	-
29.	Netherlands	90	-	1	2	3
30.	Colombia	88	-	1	2	-
31.	New Zealand	83	-	-	2	2
32.	South Korea	79	-	1	1	1
33.	Thailand	75	-	-	2	2
34.	Turkey	75	-	-	1	2
35.	Spain	72	-	-	-	-
36.	Morroco	71	-	1	-	-
37.	Mexico	69	-	-	1	2
38.	Argentina	67	-	-	1	2
39.	Cuba	67	-	-	1	1
40.	Ireland	65	-	-	1	-
41.	Bahrain	65	-	-	-	1
42.	Greece	62	-	-	1	1
43.	Finland	59	-	-	1	1
44.	Luxembourg	58	1	1	-	-
45.	Tunisia	55	-	1	1	-
46.	Mongolia	54	-	-	-	3
47.	Kuwait	53	-	-	1	1
48.	Cyprus	46	-	-	1	-
49.	Philippines	46	-	-	1	-
50.	Portugal	44	-	-	-	-
51.	Indonesia	40	-	-	-	-
52.	Macao	32	-	-	-	-
53.	Iceland	30	-	-	1	-
54.	Algeria	29	-	-	-	-