Problems and Solutions

In this issue, the Mathematical Medley starts a section called *Problems and Solutions.* The aim is to encourage more readers to participate in the intriguing process of problem solving in Mathematics. This section publishes problems and solutions proposed by readers and editors.

Readers are welcome to submit solutions to the following problems. Your solutions, if chosen, will be published in the next issue, bearing your full name and address. A publishable solution must be correct and complete, and presented in a well-organised manner. Moreover, elegant, clear and concise solutions are preferred. Solutions should reach the editors before 30 October, 1990.

Readers are also invited to propose problems for future issues. Problems should be submitted with solutions, if any. Relevant references should be stated. Indicated with an ° if the problem is original and with an * if its solution is not available.

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Problems

P18.1.1. *Proposed by Wong Ngai Ying, Chinese University of Hong Kong.*

Prove that

$$M_p(\underline{a}) \geq M_q(\underline{a}) \quad \text{for } p \geq q,$$

where $M_t(\underline{a}) = \left(\frac{1}{n}\sum_{r=1}^{n} a_r^t\right)^{\frac{1}{t}}$, for the positive numbers $a_1, a_2, \ldots, a_n$.

The following are questions in The 1990 Asian Pacific Mathematical Olympiad:

P18.1.2

In $\triangle ABC$, let D, E, F be the midpoints of BC, AC, AB respectively and let G be the centroid of the triangle.

For each value of $\angle BAC$, how many non-similar triangles are there in which $AEGF$ is a cyclic quadrilateral?

P18.1.3

Let $a_1, a_2, \ldots, a_n$ be positive real numbers, and let S_k be the sum of products of $a_1, a_2, \ldots, a_n$ taken k at a time.

Show that

$$S_k S_{n-k} \geq \binom{n}{k}^2 a_1 a_2 \ldots a_n, \quad \text{for} \quad k = 1, 2, \ldots, n-1$$

P18.1.4

Consider all triangles ABC which have a fixed base AB and whose altitude from C is a constant h. For which of these triangles is the product of its altitudes a maximum?

P18.1.5

A set of 1990 persons is divided into non-intersecting subsets in such a way that

(a) no one in a subset knows all the others in the subset;

(b) among any three persons in a subset, there are always at least two who do not know each other; and

(c) for any two persons in a subset who do not know each other, there is exactly one person in the same subset knowing both of them.

(i) Prove that within each subset, every person has the same number of acquaintances.

(ii) Determine the maximum possible number of subsets.

Note: It is understood that if a person A knows person B, then person B will know person A; an acquaintance is someone who is known. Every person is assumed to know one's self.

P18.1.6

Show that for every integer $n \geq 6$, there exists a convex hexagon which can be dissected into exactly n congruent triangles.