# Singapore Mathematical Society

## Inter-Secondary School
## Mathematical Competition 1989

Saturday, 3 June 1989 1000–1200

*Attempt as many questions as you can.*
*No calculators are allowed.*
*Circle your answers on the Answer Sheet provided.*
*Each question in Sections A, B and C carries 2, 3 and 5 marks, respectively.*

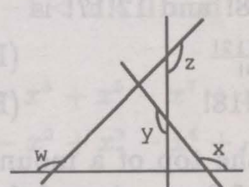
## Section A

1. Find all real $x$ which satisfy the inequality $|x^2 - 5x| > 6$.

   (A) $2 < x < 3$ (B) $x > 6$ or $x < -1$
   (C) $x > 6$ or $x < -1$ or $2 < x < 3$ (D) $-1 < x < 6$
   (E) None of the preceding

2. Four straight lines intersect as shown. The value of $x + y + z + w$ in degrees is

   (A) 360
   (B) 450
   (C) 530
   (D) 630
   (E) None of the preceding




3. The sum of the arithmetic progression 5, $4\frac{2}{7}$, $3\frac{4}{7}$, ... from the $n^{\text{th}}$ term to the $(n+6)^{\text{th}}$ term is $T$. Find the value of $n$ such that the absolute value $|T|$ of $T$ is the smallest.

   (A) 6 (B) 5 (C) 4 (D) 3 (E) 7

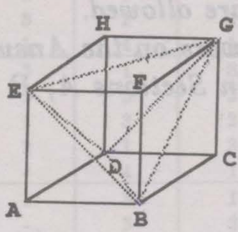
4. 35 persons per thousand have high blood pressure, 80% of those with high blood pressure drink, and 60% of those without high blood pressure drink. What percentage of drinkers have high blood pressure?

   (A) 1.2% (B) 2.9% (C) 4.6%
   (D) 6.8% (E) None of the preceding

5. Let $\alpha$ and $\beta$ be the roots of the equation $x^2-(k-2)x+(k^2+3k+5)=0$, where $k$ is a real constant. If $\alpha$ and $\beta$ are real, find the maximum value of $\alpha^2+\beta^2$.

(A) 18 (B) 19 (C) $5\frac{5}{9}$ (D) 17 (E) 20

6. The ratio of the surface area of the cube $ABCDEFGH$ to the surface area of the tetrahedron $DEGB$ is




(A) $3:1$

(B) $4:1$

(C) $\sqrt{2}:1$

(D) $\sqrt{3}:1$

(E) $2:1$

7. In $\Delta PQR$, $S$ is a point on $PQ$ so that $PR=35$, $PS=11$ and $RQ=RS=31$. The length of $SQ$ is

(A) 10 (B) 11 (C) 12 (D) 13 (E) None of the preceding

8. If $n$ is a fixed positive number and $f(x^n)=\log x$, for all $x>0$, then the value of $f(2)$ is

(A) $\log 2$ (B) $n\log 2$ (C) $2^n\log 2$ (D) $\log(2n)$ (E) $\frac{1}{n}\log 2$

9. We define $n!=n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$. The least common multiple of 10!18! and 12!17! is

(A) $\frac{18!12!}{6!}$ (B) 18!17! (C) $\frac{12!18!}{3!}$

(D) 12!18! (E) $\frac{18!17!}{6!}$

10. From the top of a mountain of height $h$ meters above sea level, the angle of depression of the horizon is $\theta$. Then the radius of the earth in meters is

(A) $\frac{h}{\sec\theta-1}$ (B) $\frac{h}{1-\cos\theta}$ (C) $\frac{h}{\operatorname{cosec}\theta-1}$

(D) $\frac{h}{1-\sin\theta}$ (E) None of the preceding

## Section B

11. The minimum value of $(x-2)(x-4)(x-6)(x-8)+10$ is

(A) $-4$ (B) $-6$ (C) $-8$ (D) 6 (E) 8

12. The exact value of $\tan 9^\circ+\cot 117^\circ-\tan 243^\circ-\cot 351^\circ$ is

(A) 2 (B) 3 (C) 4 (D) $\sqrt{2}$ (E) $\frac{9}{2}$

**13.** Let $f(x) = \frac{1+x}{1-x}$. Define $f^n(x) = f(f^{n-1}(x))$, $n \geq 2$. Then the value of $f^{1989}(1989)$ is

(A) $\frac{1}{1988}$ (B) $-\frac{1990}{1988}$ (C) 1990 (D) $\frac{1}{1989}$ (E) $-\frac{1988}{1990}$

**14.** In the figure below, $P$ is the centre of the square constructed on the hypotenuse $AC$ of the right-angled triangle $ABC$. What is the relationship between $\angle ABP$ and $\angle PBC$?

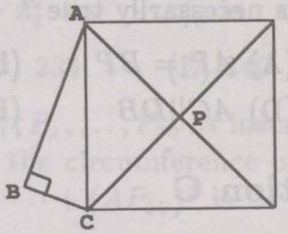
(A) $\angle ABP = \angle PBC$

(B) $\angle ABP = \frac{1}{4}\angle PBC$

(C) $\angle ABP = 2\angle PBC$

(D) $\angle ABP = \frac{1}{2}\angle PBC$

(E) None of the preceding




**15.** In the square $ABCD$, $ABCGH$, $AEFCD$, $BCDHE$ and $ABFGD$ are quarters of circles with centres at $B$, $D$, $C$ and $A$, respectively. Find $\angle ECH$.

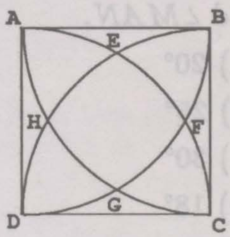
(A) 45°

(B) 30°

(C) 15°

(D) 22.5°

(E) None of the preceding




**16.** Given $x = y + y^2 + y^3 + \cdots + y^n + \cdots$, $-1 < x < 1$ and $-1 < y < 1$, expand $y$ in powers of $x$.

(A) $x + x^2 + x^3 + x^4 + \cdots$ (B) $x - x^3 + x^5 - x^7 + x^9 - \cdots$

(C) $x + 2x^2 + 3x^3 + 4x^4 + \cdots$ (D) $x - x^2 + x^3 - x^4 + \cdots$

(E) None of the preceding

**17.** If the function $f(x) = ax^2 - c$ is such that $-4 \leq f(1) \leq -1$ and $-1 \leq f(2) \leq 5$, which of the following is always true?

(A) $7 \leq f(3) \leq 26$ (B) $-4 \leq f(3) \leq 15$

(C) $-1 \leq f(3) \leq 20$ (D) $-\frac{28}{3} \leq f(3) \leq \frac{35}{3}$

(E) None of the preceding

**18.** In $\Delta ABC$, $AB = AC$, $D$ is the mid-point of $BC$, $E$ is the mid-point of $AC$. If $AD = 24$, $BE = 18$, then the area of $\Delta ABC$ is

(A) 214 (B) $124\sqrt{3}$ (C) $87\sqrt{6}$

(D) $96\sqrt{5}$ (E) None of the preceding

19. Find the largest positive integer $n$ such that $2^n$ divides the product $101 \cdot 102 \cdots 199 \cdot 200$.

(A) 95 (B) 98 (C) 100 (D) 103 (E) 105

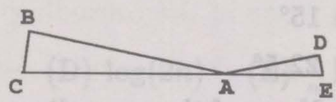
20. Suppose the area of $\Delta ABC$ is equal to the area of $\Delta ABD$ and the segments $AB$ and $CD$ intersect at a point $P$. Which of the following is necessarily true ?

(A) $AP = BP$ (B) $DP = CP$ (C) $BC || DA$
(D) $AC || DB$ (E) None of the preceding

## Section C

21. In the figure below, $ABC$ and $ADE$ are two triangles such that $AB = AC$, $AD = AE$, $\angle BAC = \angle DAE = 22°$, and the points $C$, $A$, $E$ are collinear. If $M$ is the mid-point of $CD$ and $N$ is the mid-point of $BE$, find $\angle MAN$.

(A) 20°
(B) 25°
(C) 30°
(D) 18°
(E) None of the preceding




22. If $x = \sqrt{12 - 6\sqrt{3}}$, the value of $\frac{x^4 - 6x^3 + 9x^2 - 18x + 23}{x^2 - 6x + 8}$ is

(A) $\sqrt{3}$ (B) $\frac{5}{2}$ (C) 5 (D) 10 (E) 15

23. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

(A) $\pi/2$ (B) $\pi/3$ (C) $\pi/6$ (D) $\pi/8$ (E) $\pi/4$

24. Find the sum $1 + \frac{1}{1+2} + \cdots + \frac{1}{1+2+3+\cdots 100}$.

(A) $\frac{170}{101}$ (B) $\frac{180}{101}$
(C) $\frac{190}{101}$ (D) $\frac{200}{101}$ (E) $\frac{210}{101}$

25. A positive number $p$ when divided by $n$ leaves a remainder $n - 1$, for $n = 2, 3, \ldots, 10$. Find the least possible value of $p$.

(A) 2579 (B) 2939 (C) 3779
(D) 5039 (E) None of the preceding

**26.** The number of integers from 1 to 1000 which are not divisible by 9 and 10 is

(A) 789 (B) 800 (C) 889

(D) 900 (E) None of the preceding

**27.** Suppose $x_1$, $x_2$, ..., $x_{10}$ are 10 positive integers satisfying $x_1 + x_2 + \cdots + x_{10} = 42$. The minimum value of $x_1^2 + x_2^2 + \cdots + x_{10}^2$ is

(A) 109 (B) 178 (C) 180 (D) 234 (E) 240

**28.** A regular 20-sided polygon with vertices $P_1, P_2, \ldots, P_{20}$ is inscribed in a circle of radius 1. If $A$ is a point on the circumference of the circle, then the value of $(AP_1)^2 + (AP_2)^2 + \cdots + (AP_{20})^2$ is

(A) 30 (B) 40 (C) 50 (D) 60 (E) None of the preceding

**29.** In the following long division, each $*$ represents a missing digit. Find the divisor.

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             * 7 * 0 *
      * * * ) * * * * * * * *
              * * * *
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                * * *
                * * *
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(A) 123 (B) 124 (C) 125 (D) 126 (E) None of the preceding

**30.** One side of a triangle is three times another and the perimeter is 48. If $xc$ is length of the shortest side of the triangle, then

(A) $4 < x < 5$ (B) $5 < x < 6$ (C) $6 < x < 8$

(D) $8 < x < 10$ (E) None of the preceding

—E N D—

Answers:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. c | 2. e | 3. b | 4. c | 5. a | 6. d | 7. d | 8. e |
| 9. c | 10. a | 11. b | 12. c | 13. b | 14. a | 15. b | 16. d |
| 17. c | 18. d | 19. c | 20. b | 21. e | 22. b | 23. e | 24. d |
| 25. e | 26. b | 27. b | 28. b | 29. b | 30. c | | |