# Singapore's participation in the 29th IMO

**(9-21 July, 1988, Canberra)**

Chen Chuan Chong
Department of Mathematics
National University of Singapore

The International Mathematical Olympiad (IMO) was first held in Rumania in 1959. Since then, it has become the most prestigious annual international mathematical competition for students under 20 years of age and who have not studied at a tertiary institution. The aims of IMO include:

- the discovering, encouraging and challenging of mathematically gifted schools students;
- the fostering of friendly international relations between students and their teachers; and
- the sharing of information on educational systems, mathematical syllabi and pedagogy throughout the world.

Participation in the IMO is by invitation from the host country. Each participating country may send a delegation consisting of a leader, a deputy leader and six contestants. The contestants have to go through two examinations held in two consecutive days. Each examination paper consists of 3 problems, and the time allowed for each paper is 4.5 hours.

In recent years, the number of participating countries has increased to over 40. And in 1986, for the first time, Singapore received an invitation to participate in the 29th IMO hosted by Australia from the 9th till 21st of July 1988, as part of the Australian Bicentennial Celebrations of European Settlement. With the full support of our Ministry of Education, the Singapore Mathematical Olympiad (SMO) Committee was set up in September 1986 to assist the Ministry in the selection and the training of the Singapore IMO Team. I was elected chairman of the committee, and the committee members then were: Mr. Chan Onn, Mrs. Chang Swee Tong, Prof. Koh Khee Meng, Dr. Leong Yu Kiang, Mr. Leuar Boon Char, Mr. Liew Mai Heng, Dr. Tay Yong Chiang and Mr. Yeo Kwee Poo. A selection test organized by the committee was held in the 5th of December 1986 to select 48 candidates from 599 secondary three and secondary four students. These students then received their weekly training from January 1987 till May 1987, under the supervision of Mr. Chan Onn and the late Mr. Liew Mai Heng. The training was conducted every Saturday in Hwa Chong Junior College at Bukit Timah Road. During 22nd till 27th of June 1987, an SMO Training Camp was organized at the National University of Singapore (NUS) to provide more intensive training and opportunity for the students to work with mathematicians of higher institution. Based on several tests conducted in the SMO Training Camp and the results of the Interschool Mathematical Competition 1987 organized by the Singapore Mathematical Society, 25 students were selected for further training under several lecturers at the National University. Finally, 6 students were selected in March 1988 to form the first Singapore IMO Team. They are:

| | |
|---|---|
| Chan Hock Peng | Cheong Kok Vui |
| Ngan Ngiap Teng | Lim Jing Yee |
| Deng Shao Kun | Yeoh Yong Yeow |

The team continued to receive weekly training at NUS. Besides those lecturers who were in the SMO Committee, Prof. Lee Peng Yee, Dr. Leung Pui Fai, Dr. T. R. Nanda and Dr. Tay Tiong Seng also rendered their help in giving lectures in various topics to the team. In June 1988, another training camp was organized for both the team and trainees for the 1989 IMO, just before the 29th IMO.

As I was elected the team leader of our 1988 IMO Team, I had to leave for Australia two days ahead of our IMO Team which was later accompanied by the deputy leader Dr. Tay Yong Chiang. I left Singapore on the 8th of July 1988 at about 9 p.m. and arrived at Sydney at 6 a.m. the next morning. I was received by Mr. Peter J. O'Halloran, 1988 IMO Chief Organizer. Many other team leaders of various countries also arrived at about the same time. We were taken to the Sydney YWCA, where we were formally registered before checking into our rooms. A welcome dinner was given by the New South Wales Mathematical Society in the evening at the YWCA to all team leaders and their accompanying persons. After the dinner, some 94 proposed problems with 34 short-listed were handed to all team leaders to consider for further discussions in subsequent Jury meetings.

On the 10th of July, a one-day excursion was arranged for the team leaders. The next day, all team leaders were taken by coaches to Canberra. We departed at 8 a.m. and arrived at University House of the Australian National University, Canberra, at about 12.30 p.m.. After checking in and having lunch at University House, the First Jury Meeting was immediately held at the National Academy of Science Building from 1.30 p.m. to 5.30 p.m.. There were altogether 11 Jury meetings scheduled over the next few days with the first 6 being held in the National Academy of Science Building and the rest in the Canberra College of Advanced Education (CCAE). These meetings mainly dealt with rules and regulations, the selection of the final 6 problems for the two days of competitions, the translation of these problems from the official version (which was in English) into 29 other languages, the coordination of markings and the award of medals. All the meetings were very long, as each motion had to be translated into French and sometimes German and Russian and vice versa.

Before the competition, I was kept away from our deputy leader Dr. Tay Yong Chiang who arrived with our IMO Team at Sydney on the 11th of July and at Canberra on the 14th of July. Even during the Opening Ceremony at CCAE on the 14th, all team leaders were carefully kept away from the deputy leaders and the students to avoid any unnecessary exchange of information about the competition problems. After the ceremony, team leaders were immediately sent back to University House, while the rest stayed at CCAE, in which the competition were held.

The first day of the competition was on the 15th of July. At 7.45 a.m., all team leaders were taken from University House to CCAE to answer queries by students for about half an hour when the competition started at 8.30 a.m.. The same day, all team leaders received the scripts of their respective teams at about 7.00 p.m. and started marking.

In the second day of the competition on the 16th of July, again, all team leaders (with baggage) were taken to CCAE at 7.45 a.m. to answer examination queries from 8.30 a.m. to 9.15 a.m., after which the 8th Jury Meeting was held to discuss more problems related to the coordination of the assessment of scripts. At about 10.30 a.m., rooms in CCAE hostel were allocated to the team leaders. It was only then that the team leaders met their respective deputy leaders and assessed the scripts together. We had lunch at CCAE and were then allowed to meet the team members. The coordination sessions started at 2 p.m. until 5.30 p.m.. The answers to the first questions by the Singapore Team were assessed and coordinated. Coordination sessions continued the next day at 9.00 a.m. and ended at 5.30 p.m.. The Singapore Team's scripts to the remaining 5 questions were assessed and coordinated.

On the 18th of July, the last coordination session started at 9.00 a.m.. By 1.00 p.m., the results of all countries were known. At 2.00 p.m., there was a meeting with the IMO Site Committee, to discuss the list of countries to be invited for the 1989 IMO and the eligibility of the competitors. The 9th Jury Meeting was held from 3.00 p.m. to 5.30 p.m., confirming the final scores of all participating countries and also determining the number of Gold, Silver and Bronze Medals for the competitors. It was decided that a bronze medalist must have a minimum score of 14 points, silver medalist 23 points and gold medalist 32 points. The Singapore Team won 2 silver medals (Yong Yeow and Ngiap Teng) and 2 bronze medals (Hock Peng and Kok Vui). Future host countries were also nominated in the meeting. They are Federal Republic of Germany (1989), People's Republic of China (1990), Sweden (1991), German Democratic Republic (1992), Turkey (1993), Belgium (1994), Canada (1995), Brazil/Greece (1996), United Kingdom/Hong Kong (1997). Rumania specially requested to be the host country in 1999. The 10th Jury Meeting was held from 7 p.m. to 9.30 p.m., to discuss the criterion for awarding honourable mention to non-medalists. After a lengthy discussion, it was finally decided that an honourable mention should be given to a non-medalist who had a complete solution to one of the 6 problem. It was also decided that a special award be given to Emanouil Atanassou of the Bulgarian Team, for providing the best solution.

The 11th Jury Meeting scheduled on the 19th of July was canceled. In the evening, team leaders and accompanying persons were invited to the Australian Parliament House for a cocktail party. At about 7.45 p.m., a simple ceremony was arranged to give away the medals and special award to the Bulgarian Team as they have to leave the same night. The next day, arrangements had been made for the teams of various countries to visit their respective embassies from 9.00 a.m. to 11.00 a.m.. After lunch, all participants were taken to the Canberra Theatre for the Prize Presentation Ceremony which lasted for about an hour from 2.00 p.m. to 3.00 p.m. The Prime Minister of Australia, Mr. R.J.L Hawke gave away the gold medals. There was a farewell dinner at CCAE at 7.00 p.m.. This was a last chance for competitors to exchange souvenirs and autographs. It lasted until almost 11 p.m..

On the 21st of July, most teams left for Sydney early in the morning and arrived at the Sydney International Airport at noon. The Singapore Team left Australia at 2.40 p.m. and returned to Singapore at 8.40 p.m..

In my opinion, the 1988 IMO was very well-organized. Our participation has been a wonderful experience for the whole Singapore team. It gives both the leaders and competitors a very good exposure and a very valuable chance to exchange ideas with participants from many other countries. The Singapore Team performed reasonably well and came home with 2 silver and 2 bronze medals. Our total score of 96 points ranks 18 among 49 participating countries and is the best among countries participating for the first time. I hope that this experience could be passed on to our future participants so that they could perform even better.

Finally, I would like to provide in the following (I) the problems for the 29th IMO, (II) the solutions to the problems, and (III) the list of participating countries and their overall results in the 1988 IMO.

# 29th IMO Problems

## FIRST DAY

Canberra, July 15, 1988

1. Consider two coplanar circles of radii $R$ and $r$ $(R > r)$ with the same centre. Let $P$ be a fixed point on the smaller circle and $B$ a variable point on the larger circle. The line $BP$ meets the larger circle again at $C$. The perpendicular $\ell$ to $BP$ at $P$ meets the smaller circle again at $A$ (if $\ell$ is tangent to the circle at $P$ then $A = P$).
   (i) Find the set of values of $BC^2 + CA^2 + AB^2$.
   (ii) Find the locus of the midpoint of $AB$.

2. Let $n$ be a positive integer and let $A_1, A_2, \ldots, A_{2n+1}$ be subsets of a set $B$. Suppose that
   (a) each $A_i$ has exactly $2n$ elements,
   (b) each $A_i \cap A_j$ $(1 \le i < j \le 2n+1)$ contains exactly one element, and
   (c) every element of $B$ belongs to at least two of the $A_i$.
   For which values of $n$ can one assign to every element of $B$ one of the numbers 0 and 1 in such a way that each $A_i$ has 0 assigned to exactly $n$ of its elements?

3. A function $f$ is defined on the positive integers by

$$f(1) = 1, \quad f(3) = 3, \quad f(2n) = f(n),$$
$$f(4n+1) = 2f(2n+1) - f(n),$$
$$f(4n+3) = 3f(2n+1) - 2f(n),$$

   for all positive integers $n$.
   Determine the number of positive integers $n$, less than or equal to 1988, for which $f(n) = n$.

Time: 4.5 hours

Each problem is worth 7 points

## 29th IMO Problems

SECOND DAY

Canberra, July 16, 1988

4. Show that the set of real numbers $x$ which satisfy the inequality

$$\sum_{k=1}^{70} \frac{k}{x-k} \geq \frac{5}{4}$$

is a union of disjoint intervals, the sum of whose length is 1988.

5. $ABC$ is a triangle right-angled at $A$, and $D$ is the foot of the altitude from $A$. The straight line joining the incentres of the triangle $ABD$, $ACD$ intersects the sides $AB, AC$ at the points $K, L$ respectively. $S$ and $T$ denote the areas of the triangles $ABC$ and $AKL$ respectively. Show that $S \geq 2T$.

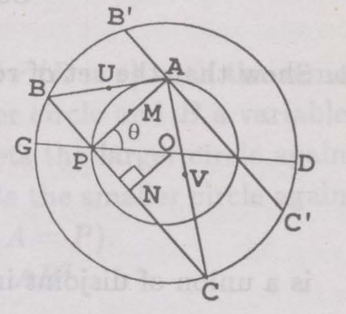
6. Let $a$ and $b$ be positive integers such that $ab+1$ divides $a^2+b^2$. Show that $\dfrac{a^2+b^2}{ab+1}$ is the square of an integer.

Time: 4.5 hours

Each problem is worth 7 points.

## Solutions of the problems of the 1988 IMO examinations

**(1.)** Let $\angle OPA = \theta$, $GD =$ the diameter through $P$, $M$ the midpoint of $PA$ and $N$ that of $BC$. Let $U$ be the midpoint of $AB$ and $V$ the midpoint of $AC$.




The sum

$$
\begin{aligned}
S &= BC^2 + CA^2 + AB^2 \\
&= (BP + PC)^2 + PC^2 + PA^2 + BP^2 + PA^2 \\
&= 2(PA^2 + PB^2 + PC^2 + BP \cdot PC) \qquad (1)
\end{aligned}
$$

We have

$$
\begin{aligned}
PA &= 2r\cos\theta \\
BP &= BN - PN = \sqrt{R^2 - r^2\cos^2\theta} - r\sin\theta \\
PC &= PN + NC = BN + PN = \sqrt{R^2 - r^2\cos^2\theta} + r\sin\theta \\
BP \cdot PC &= GP \cdot PD = R^2 - r^2.
\end{aligned}
$$

Substituting in (1) one has

$$
\begin{aligned}
S &= 2[4r^2\cos^2\theta + 2(R^2 - r^2\cos^2\theta + r^2\sin^2\theta) + R^2 - r^2] \\
&= 6R^2 + 2r^2.
\end{aligned}
$$

The sum is constant, and so independent of $\theta$.

The parallel to $BC$ through $A$ meets the larger circle in $B'$ and $C'$, which are vertices of the rectangles $BPAB'$ and $CPAC'$. The midpoint $U$ of the diagonal $BA$ is also the midpoint of the diagonal $PB'$, and $\overline{PU} = \frac{1}{2}\overline{PB'}$. Similarly, $\overline{PV} = \frac{1}{2}\overline{PC'}$.

Since $B'$ and $C'$ described the same circle $(O, R)$, the locus of $U$ and $V$ is unique which is the image of the circle $(O, R)$ under dilatation $H(P; \frac{1}{2})$, i.e., the circle $(O', \frac{1}{2}R)$ where $O'$ is the mid-point of $OP$.

**(2.)** We prove that such an assignment is possible if and only if $n$ is even.

1. To begin, we show that conditions (a) – (c) imply a strengthened version of (c), viz.

($c^*$) Every element of $B$ belongs to exactly two of the $A_i$.

First note that

$$A_j = \bigcup_{\substack{i=1 \\ i \neq j}}^{2n+1} (A_i \cap A_j), \quad j = 1, ..., 2n+1. \qquad (*)$$

The inclusion $\supseteq$ is trivial, and the reverse inclusion follows from (c).

Secondly, suppose, contrary to $(c^*)$, $a \in A_1 \cap A_2 \cap A_3$ say. Then by (b), $(A_1 \cap A_2) \cup (A_1 \cap A_3)$ and each $(A_1 \cap A_i)$, $i > 3$, contain only one element. Hence, by $(*)$, $A_1$ contains at most $2n-1$ members, contradicting (a).

2. Next we show that if 0's and 1's can be assigned to the elements of $B$ in the required manner, then $n$ must be even. We define a $2n \times 2n$ table as follows: in row $i$, column $j$, we put the number assigned to the (unique) element of $A_i \cap A_j$ if $i \neq j$, and the number assigned to the (unique) element of $A_i \cap A_{2n+1}$ if $i = j$. By assumption and $(c^*)$ each row contains $n$ 0's, and so the whole table contains $2n^2$ 0's, which is an even number. Since the table is symmetric about the main diagonal, there is an even number of 0's off the main diagonal. Hence there is an even number of 0's on the main diagonal. But the numbers on the main diagonal are the numbers assigned to the elements of $A_{2n+1}$ and so $n$ of them are 0's. Hence $n$ is even.

3. Finally we show that if $n$ is even then the required assignment of 0's and 1's is possible. Let $T$ be the table defined by

$$T = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix},$$

and for $n = 2k$, let $U$ be the $2n \times 2n$ table defined by

$$U = \underbrace{\begin{pmatrix} T & T & \cdots & T \\ \vdots & \vdots & \cdots & \vdots \\ T & T & \cdots & T \end{pmatrix}}_{k \text{ times}}.$$

Then $U$, interpreted as in 2., gives an assignment as required.

**(3.)** We find

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17... |
| $f(n)$: | 1 | 1 | 3 | 1 | 5 | 3 | 7 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 | 1 | 17... |

It appears that $f(2^k) = 1$, $f(2^k - 1) = 2^k - 1$, $f(2^k + 1) = 2^k + 1$ which suggest a connection with binary expansions. We soon conjecture that

$f(n) =$ the number obtained by reversing the binary expansion of $n$ (ignoring any initial zeros which may result).

The proof is by induction. Since $f(2n) = f(n)$, only odd numbers need to be considered.

If $n$ is of the form

$$4m + 1 = \sum_{j=0}^{k} \epsilon_j 2^j, \qquad \epsilon_0 = 1, \quad \epsilon_1 = 0$$

then

$$m = \sum_{j=2}^{k} \epsilon_j 2^{j-2}, \qquad 2m + 1 = 1 + \sum_{j=2}^{k} \epsilon_j 2^{j-1}.$$

By induction

$$f(2m+1) = 2^{k-1} + \sum_{j=2}^{k} \epsilon_j 2^{k-1-(j-1)} = 2^{k-1} + \sum_{j=2}^{k} \epsilon_j 2^{k-j},$$

$$f(m) = \sum_{j=2}^{k} \epsilon_j 2^{k-j}.$$

Hence

$$\begin{aligned} 2f(2m+1) - f(m) &= 2^k + 2\sum_{j=2}^{k} \epsilon_j 2^{k-j} - \sum_{j=2}^{k} \epsilon_j 2^{k-j} \\ &= 2^k + \sum_{j=2}^{k} \epsilon_j 2^{k-j} = \sum_{j=0}^{k} \epsilon_j 2^{k-j}, \end{aligned}$$

as required. If $n$ is of the form

$$4m + 3 = \sum_{j=0}^{k} \epsilon_j 2^j, \qquad \epsilon_0 = 1, \quad \epsilon_1 = 1$$

then, as before

$$m = \sum_{j=2}^{k} \epsilon_j 2^{j-2}, \qquad 2m + 1 = 1 + \sum_{j=2}^{k} \epsilon_j 2^{k-j}.$$

Hence, using the same calculation as before

$$3f(2m+1) - 2f(m) = 2^k + 2^{k-1} + \sum_{j=2}^{k} \epsilon_j 2^{k-j} = \sum_{j=0}^{k} \epsilon_j 2^{k-j},$$

as required. Thus the conjecture is verified.

So we have to count the integers $n$, $1 \le n \le 1988$, which have *palindromic* binary expansions. Now the number of $2m$-digit binary palindromes is $2^{m-1}$ = the number of $(2m-1)$-digit palindromes. We have $2^{10} < 1988 < 2^{11} = 2048$ and the number of palindromes < 2048 is

$$1+1+2+2+4+4+8+8+16+16+32=94.$$

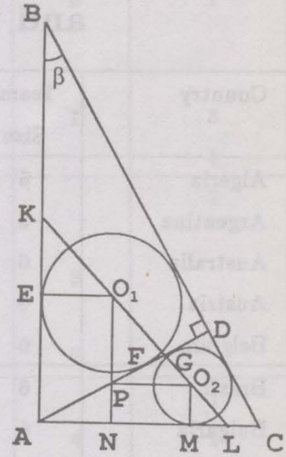
In fact, $1988 = (11111000100)_2$ and there are only two 11-digit palindromes exceeding this, so the required number is 92.

**(4.)** It is clear from a graph that the set $S$ in the question is a union of intervals of the form $(i, x_i]$, $i = 1, 2, ..., 70$, where $i < x_i < i+1 < x_{i+1}$, $i = 1, 2, ..., 69$ and the $x_i$'s are the roots of the polynomial

$$5\prod_{j=1}^{70}(x-j)-4\sum_{k=1}^{70}k\prod_{j\neq k}^{70}(x-j).$$

The sum of the roots of this polynomial is $\sum_{j=1}^{70} j + (4/5)\sum_{k=1}^{70} k$. Hence $|S| = \sum_{i=1}^{70}(x_i - i) = (4/5)\sum_{k=1}^{70} k = 1988$, as required.

**(5.)** Denote $AB = c, AC = b, BC = a$ and $AD = h$, the circle inscribed in $\triangle ABD$ by $C_1$, and that inscribed in $\triangle ADC$ by $C_2$. Let $O_1, O_2$ be the centres and $r$, $R$ be the radii of $C_1$ and $C_2$ respectively, $E$ and $F$ the points of contact of $C_1$ with $AB$ and $AD$ respectively, and $G$ and $M$ the points of contact of $C_2$ with $AD$ and $AC$ respectively. Let $O_1N$ be perpendicular to $AC$ and $O_2P$ perpendicular to $NO_1$.




Then

$$O_1N = EA = AF = h - r$$
$$O_1P = O_1N - PN = O_1N - O_2M = h - r - R$$
$$O_2P = MN = AM - AN = AG - r = h - R - r.$$

Hence $PO_1 = PO_2$ and thus $\angle O_1O_2P = 45°$. Therefore $\angle O_2LM = 45°$ and thus $ML = O_2M = R$. Consequently $AL = AM + ML = AG + R = h - R + R = h$. Similarly $AK = h$. Hence

$$\frac{S}{T} = \frac{ah}{h^2} = \frac{a}{h} = \frac{a^2}{ah} = \frac{a^2}{bc} = \frac{b^2+c^2}{bc} \ge 2.$$

(6.) Suppose $\frac{a^2+b^2}{ab+1} = k$, a positive integer. Then

$$a^2 - kab + b^2 = k \qquad (1)$$

We assume henceforth that $k$ is not a square. Every solution of (1) has $a, b > 0$ or $a, b < 0$ (Clearly, $ab \neq 0$ and if $ab < 0$, $a^2 - kab + b^2 > k$.)

Consider a solution $(a, b)$ of (1) with $a \geq b > 0$ and $a$ minimal. Note that $b < a$, since if $b = a$, we have $(2-k)a^2 = k$, but the left-hand-side is non-positive. Consider (1) as a quadratic in $a$. It has two roots, $a$ and $a_1$. We have $a + a_1 = kb$, so $a_1$ is an integer. Since $b > 0$, $a_1 > 0$. Also, $aa_1 = b^2 - k$, so $a_1 = (b^2 - k)/a < (a^2 - 1)/a < a$.

The pair $(a_1, b)$ satisfies (1) and $a_1 > 0$, $b > 0$, $a_1 < a$, $b < a$, which contradicts the minimality, and we are finished.

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## List of Participating Countries in the 29th IMO and the Distribution of Awards

| Country | Team Size | Score | Medal | | | Honourable Mention |
|---|---|---|---|---|---|---|
| | | | Gold | Silver | Bronze | |
| Algeria | 5 | 42 | | 1 | | 1 |
| Argentina | 3 | 23 | | | | 1 |
| Australia | 6 | 100 | 1 | | 1 | 1 |
| Austria | 6 | 110 | 1 | 1 | 1 | 1 |
| Belgium | 6 | 76 | | | 3 | 1 |
| Brazil | 6 | 39 | | | | 2 |
| Bulgaria | 6 | 144 | | 4 | 2 | |
| Canada | 6 | 124 | 1 | 1 | 2 | 1 |
| Columbia | 6 | 66 | | | 3 | |
| Cuba | 6 | 35 | | | | 1 |
| Cyprus | 6 | 21 | | | | 1 |
| Czechoslovakia | 6 | 120 | | 2 | 2 | 1 |
| Ecuador | 1 | 1 | | | | |
| Finland | 6 | 65 | | | 2 | 1 |
| France | 6 | 128 | 1 | 1 | 3 | 1 |

| Country | Team | Score | Medal | | | Honourable |
|---|---|---|---|---|---|---|
| | Size | | Gold | Silver | Bronze | Mention |
| Federal Republic of Germany | 6 | 174 | 1 | 4 | 1 | |
| German Democratic Republic | 5 | 145 | 1 | 4 | | |
| Greece | 6 | 65 | | | 1 | 3 |
| Hong Kong | 6 | 68 | | | 2 | 1 |
| Hungary | 6 | 109 | | 2 | 2 | 1 |
| Iceland | 4 | 37 | | | 1 | |
| Indonesia | 3 | 6 | | | | |
| Iran | 6 | 86 | | 1 | 3 | |
| Ireland | 6 | 30 | | | | |
| Israel | 6 | 115 | 1 | | 4 | 1 |
| Italy | 4 | 44 | | | 1 | 1 |
| Kuwait | 6 | 23 | | | | |
| Luxembourg | 3 | 64 | | 1 | 2 | |
| Mexico | 6 | 40 | | | 1 | 2 |
| Morocco | 6 | 62 | | | 2 | 1 |
| Netherlands | 6 | 85 | | | 3 | 1 |
| Norway | 6 | 33 | | | | |
| New Zealand | 6 | 47 | | 1 | | |
| Peru | 6 | 55 | | | 1 | 3 |
| Philippines | 5 | 29 | | | | 1 |
| Poland | 3 | 54 | | 1 | | 2 |
| Peoples Republic of China | 6 | 201 | 2 | 4 | | |
| Republic of Korea | 6 | 79 | | | 3 | |
| Rumania | 6 | 201 | 2 | 4 | | |
| Singapore | 6 | 96 | | 2 | 2 | |
| Spain | 6 | 34 | | | | 1 |
| Sweden | 6 | 115 | 1 | | 4 | 1 |
| Tunisia | 4 | 67 | | | 3 | |
| Turkey | 6 | 65 | | | 3 | |
| United Kingdom | 6 | 121 | | 3 | 2 | |
| United States | 6 | 153 | | 5 | 1 | |
| USSR | 6 | 217 | 4 | 2 | | |
| Peoples Republic of Vietnam | 6 | 166 | 1 | 4 | | |
| Yugoslavia | 6 | 92 | | | 4 | 1 |