Singapore Mathematical Olympiad
Selection Test

Friday, 2 December 1988 0930 – 1230

Question Sheet

Instructions. This test consists of FORTY questions. Attempt as many questions as you can. Circle only ONE answer to each question on the Answer Sheet provided. Any question with more than one answer circled will be disallowed. There is no penalty for a wrong answer.

Each question carries an equal number of marks.

No working need to be shown on the Answer Sheet.

No calculators of any sort are allowed.

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1. Let n be the number of ordered pairs (x, y) of positive integers satisfying the equation

$$3x + 2y = 881.$$

Then, we have

(a) $n \leq 100$ (b) $101 \leq n \leq 115$
(c) $116 \leq n \leq 130$ (d) $131 \leq n \leq 145$
(e) $n \geq 146$

2. Given that the following 7-digit positive integer

m	2	4	n	6	2	7

is a multiple of 99, then the value of $|m - n|$ is

(a) < 2 (b) $= 2$ (c) $= 3$ (d) $= 4$ (e) > 4

3. Two given positive real numbers x, y are such that $x > y$, $xy = 250$ and

$$(\log x - \log 5)(\log y - \log 5) = -\frac{15}{4}.$$

Then, the number of digits in the integer part of x is

(a) < 2 (b) 2 (c) 3 (d) 4 (e) > 4

4. Let x denote the number of positive even factors of 30030. Then

(a) $x \leq 25$ (b) $26 \leq x \leq 27$
(c) $28 \leq x \leq 29$ (d) $30 \leq x \leq 31$
(e) $x \geq 32$

5. Let $\mathbf{R}$ be the set of real numbers and let $f : \mathbf{R} - \{0\} \to \mathbf{R}$ be a function such that

$$3f(x) - f(\tfrac{1}{x}) = 4x,$$

for all nonzero real numbers x. Let A denote the set of all nonzero real numbers t such that

$$f(t) = f(t+1).$$

Then

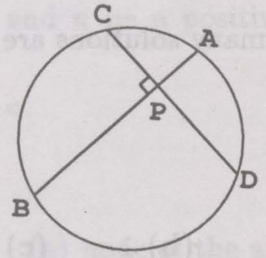
(a) A is empty
(b) A consists of one real number
(c) A consists of 2 real numbers
(d) A contains infinitely many real numbers
(e) none of the above

6. Let α denote the sum of those positive integers less than 300 which are divisible by 6 but not divisible by 8. Then α is equal to

(a) 5478 (b) 5480 (c) 5482
(d) 5484 (e) none of the above

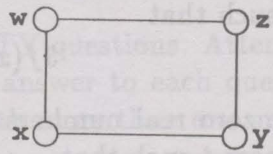
7. As shown in the figure below, two perpendicular chords AB and CD of a circle meet at P. If $AP = 1$, $CP = 2$ and $PD = 3$, then the radius r of the circle is equal to

(a) $\frac{3\sqrt{5}}{2}$
(b) $\frac{5\sqrt{2}}{2}$
(c) $\frac{3\sqrt{2}}{2}$
(d) $2\sqrt{2}$
(e) none of the above



8. The following configuration consists of 4 vertices w, x, y, z and 4 edges wx, xy, yz, zw. We wish to colour each vertex by a colour so that two vertices are coloured by distinct colours if they are joined by an edge. Suppose that there are 10 distinct colours available. Let n denote the number of different colourings of the configuration. Then

(a) $n \leq 5000$
(b) $5001 \leq n \leq 5600$
(c) $5501 \leq n \leq 6000$
(d) $6001 \leq n \leq 6500$
(e) $n \geq 6501$



9. If the function f is such that $f(2) = 2$ and $f(m+n) = f(m)f(n)$, then $f(10)$ is equal to

(a) 5 (b) 10 (c) 32 (d) 51 (e) none of the above

10. Let $x = 1 - \frac{1}{10} + \frac{1}{100} - \frac{1}{1000} + \frac{1}{10000} - \cdots$. Then

(a) $0.88 \leq x < 0.89$ (b) $0.89 \leq x < 0.90$
(c) $0.90 \leq x < 0.91$ (d) $0.91 \leq x < 0.92$
(e) none of the above

11. If $x + y < \frac{1}{2}$ and $x^2 + y^2 < 2$, then

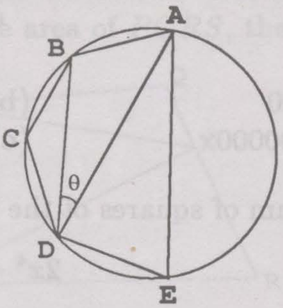
(a) $-\sqrt{2} < x < \frac{1}{4}(1+\sqrt{15})$ (b) $\frac{1}{4}(1-\sqrt{15}) < x < \frac{1}{4}(1+\sqrt{15})$
(c) $-1 < x < \frac{1}{4}(1-\sqrt{15})$ (d) $0 < x < \frac{1}{4}(1+\sqrt{15})$
(e) none of the above

12. How many solutions are there to the equation

$$x^2 + \pi^2 \cos x = 0?$$

(a) 0 (b) 1 (c) 2 (d) 3 (e) none of the above

13. As shown in the figure below, A, B, C, D and E are 5 points on the circle such that $AB = BC = CD = DE = 1$. Let $AE = x$ and $\angle ADB = \theta$. Then, we have:



(a) $x = 4\cos\theta\cos 2\theta$
(b) $x = 2\cos\theta\cos 2\theta$
(c) $x = 4\sin\theta\sin 2\theta$
(d) $x = 2\sin\theta\sin 2\theta$
(e) none of the above

14. If $a > 1$, then $\log_{\sqrt{2}} a + \log_a \sqrt{2}$ is

(a) less than 1
(b) equal to 1
(c) between 1 and 2
(d) always greater than 2
(e) none of the above

15. The integer part of the number

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{990025}}$$

is

(a) 1987 (b) 1988 (c) 1989 (d) 1990 (e) none of the above

16. The sum of the ages of n monkeys is 1988 years. If the product of the ages is to be a maximum, then the value of n must be

(a) 2 (b) 1988 (c) 600 (d) 663 (e) none of the above

17. Let x, y, z be distinct positive integers and n be a positive integer such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = n.$$

The value of n must be

(a) 3 (b) 6 (c) 9 (d) 12 (e) none of the above

18. The value of the sum

$$\sum_{k=1}^{100} \frac{k \cdot k!}{100^k} \binom{100}{k}$$

is

(a) 100 (b) 1000 (c) 10000
(d) 100000 (e) none of the above

19. The sum of squares of the roots of the equations

$$2x^4 - 8x^3 + 6x^2 - 3 = 0$$

is

(a) 5 (b) 10 (c) 15 (d) 20 (e) 26

20. Given that all the roots of the equation

$$x^4 - 4x^3 + ax^2 + bx + 1 = 0$$

are positive, then the values of a and b are

(a) $a = 2, b = 3$ (b) $a = 5, b = -5$
(c) $a = 6, b = -4$ (d) $a = -2, b = 2$
(e) none of the above

21. n points are given on the circumference of a circle, and the chords determined by them are drawn. If no three chords have a point in common, how many triangles are there all of whose sides are segments of the chords and all of whose vertices lie inside the circle?

(a) $\binom{n}{3}$ (b) $\binom{n}{4}$ (c) $\binom{n}{5}$ (d) $\binom{n}{6}$ (e) none of the above

22. Let $f(x) = x^4 + x^3 + x^2 + x + 1$. What is the remainder when $f(x^5)$ is divided by $f(x)$?

(a) 5 (b) 10 (c) x (d) $x^2 + 1$ (e) none of the above

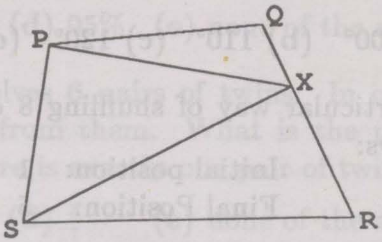
23. Suppose that

$$(1 - 3x + 3x^2)^{743}(1 + 3x - 3x^2)^{744} = a_0 + a_1 x + a_2 x^2 + \dots.$$

Then $a_0 + a_1 + a_2 + \cdots$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3 (e) none of the above

24. Let X be any point on the side QR of the quadrilateral $PQRS$ (see figure below). A line is drawn through Q parallel to PX, and another line is drawn through R parallel to SX. These two lines meet at Y. If A is the area of $\triangle PSY$ and B is the area of $PQRS$, then



(a) $A = B$
(b) $A < B$
(c) $A > B$
(d) $A = 2B$
(e) none of the above

25. The value of $\frac{1}{\log_2 36} + \frac{1}{\log_3 36}$ is

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) none of the above

26. The product

$$(1+0.5)(1+(0.5)^2)(1+(0.5)^4)\ldots(1+(0.5)^{2^n})\ldots$$

is equal to

(a) infinity (b) 10 (c) 5 (d) 2 (e) 3

27. Find the range of a real constant a for which the equation

$$x^3 - 3x + a = 0$$

has 3 distinct real roots.

(a) all real numbers
(b) empty set
(c) $-\frac{1}{3} < a < \frac{1}{3}$
(d) $-1 < a < 1$
(e) $-3 < a < 3$

28. Three numbers a_1, a_2, a_3 are chosen at random from 1,2,3,4 and 5 with $a_1 < a_2 < a_3$. Then a_1 white balls, a_2 black balls and a_3 red balls are placed in an urn, from which one ball is drawn at random. What is the probability that the ball drawn is red?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$ (e) none of the above

29. Triangle ABC has a right angle at B. If a point D in the triangle is chosen so that

$$\angle DAC = \angle DBA = \angle C = 20°,$$

then $\angle ADB$ is

(a) 100° (b) 110° (c) 120° (d) 130° (e) none of the above

30. A particular way of shuffling 8 cards would rearrange the cards as follows:

Initial position:	1	2	3	4	5	6	7	8
Final Position:	5	2	8	6	4	1	3	7

Thus, after the shuffle, the card that is initially on top would become the sixth card, the second card would remain second, and so forth. What is the minimum number of shuffles needed to get the cards back to their original arrangement?

(a) 12 (b) 24 (c) $\binom{8}{4}$ (d) 7! (e) 8!

31. In the game Ottol, one buys a ticket and selects 6 numbers out of the 44 numbers $1, 2, 3, \ldots, 44$. Subsequently, 6 of the 44 numbers are drawn as the winning numbers. A consolation prize is awarded to a selection that *does not* match any of the six winning numbers. In order to be certain of receiving a consolation prize, what is the minimum number of tickets one must buy?

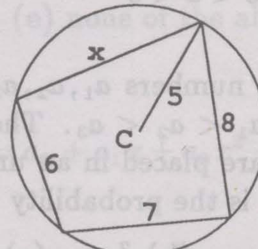
(a) 6 (b) 7 (c) 8 (d) 9 (e) 10

32. If $x = y + \frac{1}{y+\frac{1}{y+\cdots}}$ and $y = x - \frac{1}{x-\frac{1}{x-\cdots}}$, then x is equal to

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{2}-1$
(d) $\sqrt{2}+1$ (e) none of the above

33. If C is the centre of the circle shown in the following figure, then x is equal to

(a) 5
(b) 6
(c) 7
(d) 8
(e) none of the above



34. In a certain country, 80% of all married women are working, and 75% of all married women are over 35 years old. Among working married woman, 70% are over 35 years old. What proportion of nonworking married women are over 35 years old?

(a) 15% (b) 19% (c) 81% (d) 95% (e) none of the above

35. A psychology experiment involves 6 pairs of twins. In one test, 5 persons are randomly chosen from them. What is the probability that, among the 5 persons, there is exactly one pair of twins?

(a) $\frac{2}{11}$ (b) $\frac{20}{99}$ (c) $\frac{1}{5}$ (d) $\frac{3}{7}$ (e) none of the above

36. How many 0's are there between the decimal point and the first nonzero digit in the decimal representation of 0.5^{100}?

(a) 30 (b) 33 (c) 36 (d) 39 (e) none of the above

37. In how many ways can you choose 4 numbers out of 1, 2, 3, . . . , 20 so that their sum is divisible by 4?

(a) 20 (b) 620 (c) 970 (d) 1000 (e) none of the above

38. Let f be a real polynomial function such that $f(x^2+1) = x^4+5x^2+3$. Then $f(x^2-1)$ is equal to

(a) x^4+5x^2+1 (b) x^4+x^2-3
(c) x^4-5x^2+1 (d) x^4+x^2+3
(e) none of the above

39. The unit digit of $3^{1001} \times 7^{1002} \times 13^{1003}$ is

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

40. If $xy = 10$, $yz = 20$ and $zx = 30$, then $x^2+y^2+z^2$ is equal to

(a) $\frac{200}{3}$ (b) $\frac{211}{3}$ (c) $\frac{245}{3}$ (d) $\frac{489}{6}$ (e) none of the above

Answers

1. (e)	2. (b)	3. (d)	4. (e)	5. (c)	6. (a)	7. (b)	8. (e)
9. (c)	10. (c)	11. (a)	12. (e)	13. (a)	14. (e)	15. (a)	16. (d)
17. (e)	18. (a)	19. (b)	20. (c)	21. (d)	22. (a)	23. (b)	24. (a)
25. (d)	26. (d)	27. (e)	28. (e)	29. (b)	30. (a)	31. (b)	32. (e)
33. (c)	34. (d)	35. (b)	36. (a)	37. (c)	38. (b)	39. (e)	40. (c)