The Chinese Rod Numeral Legacy and its Impact on Mathematics*

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First, let me explain the Chinese rod numeral system.

Since the Warring States period (480 B.C. to 221 B.C.) to the 17th century A.D. the Chinese used a bundle of straight rods for computation. These rods, usually made from bamboo though they could be made from other materials such as bone, wood, iron, ivory and jade, were used to form the numerals 1 to 9 as follows:

1	2	3	4	5	6	7	8	9
𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨

Note that for numerals 6 to 9, a horizontal rod represents the quantity five.

A numeral system which uses place values with ten as base requires only nine signs. Any numeral of such a system is formed from among these nine signs which are placed in specific place positions relative to each other. Our present numeral system, commonly known as the Hindu-Arabic numeral system, is based on this concept; the value of each numeral determines the choice of digits from the nine signs 1, 2, ..., 9 and their place positions. The place positions are called units, tens, hundreds, thousands, and so on, and each is occupied by at most one digit.

The Chinese rod system employs the same concept. However, since its nine signs are formed from rod tallies, if a number such as 34 were represented as 𝍢𝍣, this would inevitably lead to ambiguity and confusion. To

* Text of Presidential Address delivered at the Society's Annual General Meeting on 20 March 1987.

deal with this, the ancient Chinese had two sets of rod notations, one for digits in odd (units, hundreds, ten thousands, etc.) positions, the other for digits in even (tens, thousands, hundred thousands, etc.) positions. By rotating the rods of a digit in one position through a right angle such that the horizontal rods become vertical and the vertical rods become horizontal, the digit would be transformed into its counterpart in the other position. The nine signs displayed above are used in odd positions. Their counterparts in even positions are as follows:

1	2	3	4	5	6	7	8	9
𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱

Thus, a number such as 7614 appears as 𝍯𝍥𝍩𝍣. In the same way, the number 703 appears as 𝍦 𝍢 with a blank space in the tens place position and 7003 as 𝍯   𝍢 with blank spaces in the tens and hundreds positions.

The Hindu-Arabic system initially had a similar representation so that in a number such as 703, the tens position was left blank without any symbol. Since blank spaces could lead to ambiguity in the value of a written numeral, a notation in the form of a circle or a dot was later introduced instead of a blank space. This notation gradually stabilized to the zero symbol that we are now familiar with. The zero concept is thus a natural outcome of a numeral system which uses place values. The Babylonian numeral system, which uses place values with sixty as base, also has a blank space to represent the concept of zero. As for the Mayan numerals, which can be said to be a vigesimal place value system, a symbol which looks like an eye is used to denote zero. A numeral system which does not have the place value feature would not require the concept of zero. For instance, 703 is written as $\psi\gamma$ in Greek alphabetic numerals, where $\psi = 700$ and $\gamma = 3$; the spacing and place positions of these two numerals do not affect the value of the number.

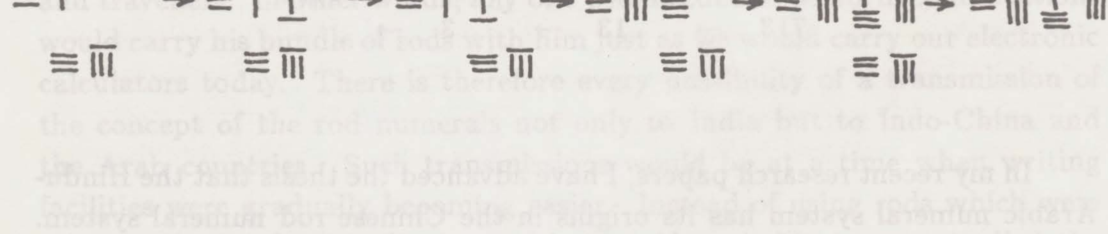
What we now call arithmetic which is taught in our schools was known to the ancient Chinese; they did their computations with rod numerals. The ***Jiu zhang suanshu*** (Nine chapters of the mathematical art) is the earliest existing Chinese treatise which has problems involving common fractions, proportions, rule of three, rule of false position, areas and volumes, extractions of square and cube roots, solutions of linear equations and other topics. This book is a compilation of such mathematical material, and it is generally accepted that it was written between 100 B.C. and 100 A.D. Some parts of the book are probably earlier than this period.

The procedures for addition, subtraction, multiplication and division using rod numerals are not found in the *Jiu zhang suanshu.* They were assumed to be known as the standard of the book was beyond these elementary arithmetical operations. The *Sun Zi suanjing* (The mathematical classics of Master Sun) written about 400 A.D. has a detailed description on multiplication and division. I shall show the steps of these procedures through examples.

Multiplication: 76×38

76		6		6		6				
21	→	266	→	266	→	284	→	2888	→	2888
38		38		38		38		38		

In rod numerals, the operations appear as follows:



Division : 2976 ÷ 8

		3		3		3		37
2976	→	2976	→	576	→	576	→	576
8		8		8		8		8

	37		37		372		372		372
→	16	→	16	→	16	→		→	
	8		8		8		8		

Note that in both multiplication and division, the operations begin from the left. The multiplier or divisor occupying the third row is shifted form left to right and finally removed. Both procedures require knowledge of the multiplication tables. The procedures of addition and subtraction are not given but they can be inferred from those of multiplication and division. From my deductions, the steps are as follows:

Addition: 5639 + 713

5639		6339		6349		6352
713	→	13	→	3	→	

Subtraction : 6352 − 713

6352		5652		5642		5639
713	→	13	→	3	→	

In my recent research papers, I have advanced the thesis that the Hindu-Arabic numeral system has its origins in the Chinese rod numeral system. Let me briefly discuss some of the main reasons that I have put forward.

We have seen that the rod numeral system and the Hindu-Arabic numeral system are based on the same concept; that is, both systems require only nine signs since they use place values with ten as base. A study of all known numeral systems preceding the Hindu-Arabic numerals reveals that the Chinese rod system is the only one that is based on this concept.

Up till now, it is generally assumed that the Hindu-Arabic system has its origins in India. However, there is no substantial evidence to justify this. The first epigraphic evidence found in India was in 595 A.D.; this was in the form of a single date inscription. The hypothesis of an Indian origin is generally based on two factors. The earliest books describing the Hindu-Arabic numerals were from Islam (and not India), and the authors called the numerals Indian. The first of these books was written by al-Khwarizmi around 825 A.D. The second factor is that the shapes of the nine signs show some resemblance with the first nine numerals of the ancient Brahmi numeral system. However, it should be pointed out that initially the nine signs of the Hindu-Arabic numerals did not have a consistent form; their shapes varied from place to place. The Brahmi and the Kharosti numerals are the two main ancient numeral systems of India. They use the same fundamental concepts as the Egyptian hieratic and the Greek alphabetic numerals. These systems do not use place values; they require notations or signs for numbers 1,2, ... 9; 10, 20 ... 90; 100, 200, ..., 900; 1000, 2000, ... 9000; and so on, so that as numbers become larger more signs have to be introduced.

While no one knows how the Hindu-Arabic system originates in India, on the other hand, there is strong evidence of a transmission of the concept of the rod system to India. In China, rod numerals were used for computations for a continuous period of two thousand years; in fact the rod system fell into disuse only some three hundred years ago. This mechanism was employed not only by mathematicians but was also commonly used among officials, traders and travellers. In other words, any one who needed to perform computations would carry his bundle of rods with him just as we would carry our electronic calculators today. There is therefore every possibility of a transmission of the concept of the rod numerals not only to India but to Indo-China and the Arab countries. Such transmissions would be at a time when writing facilities were gradually becoming easier. Instead of using rods which were alien to their cultures, these countries would probably have embodied the concept in a written form. The nine signs would then be in notations which they were familiar with.

The Chinese had their own written numerals and the rod system was primarily for computation. The rods were used for reckoning in the same way as the abaci of medieval Europe. However, while the latter did not go beyond very simple calculations, the rod system was the basis for the development of arithmetic and algebra in China for over two millenia. Why do peoples of different countries wish to adopt a new numeral system when they are used to their own numerals? In ancient and medieval times, the most compelling reason would be that the rod system had revealed significant facilities for computations which were absent in their own numeral systems. The adoption and assimilation of such a concept would inevitably be very slow. Even when the Hindu-Arabic system was introduced into Europe, it took four hundred years before this easily assimilatable written package was adopted and accepted as superior to Europe's established numeral systems.

I shall now compare the Chinese methods of the four fundamental operations of arithmetic described above with methods used in the earliest existing Arabic texts on Hindu-Arabic numerals. The procedures for addition, subtraction, multiplication and division as given in the Latin translation of al-Khwarizmi's book, in the first methods of al-Uqlīdisī's ***Kitāb al-fusūl fī al-hisāb al-hindī*** (952 A.D) and in Kushyar ibn Labban's ***Kitāb fī usūl hisāb al-hind*** (c. 1000) are the same as the Chinese methods which I have shown. While these procedures are a natural outgrowth of the rod system, they are actually not suitable for written numerals. Later, other procedures were devised that were more appropriate for a written system. Procedures for multiplication and division are mere conventions, and the fact that the same procedures were recorded in books of two different civilizations over four hundred years apart is a remarkable phenomenon.

Finally, let me give you a very brief account of the Hindu-Arabic numerals in Europe. During the Dark Ages, the peoples of Europe had few ways of reckoning beyond finger-counting. From the 9th century onwards, al-Khwarizmi and several Arabic writers wrote their treatises on arithmetic. Europe was at the initial stage of a revival of learning and scholars there were anxious to learn from the superior civilization of Islam. From the middle of the tenth century onwards, there was a subtle infiltration of knowledge and ideas to the West. The most important and lasting of these contributions from the Arab world is undoubtedly the Hindu-Arabic numeral system.

We now call the methods of computing with these numerals arithmetic, but when the system was introduced into Europe, the mode of reckoning was known as *algorism*. This word is derived from a Latin transliteration of the name of the Arabic author, al-Khwarizmi. His work, translated into Latin in the 12th century, was the first on the new system of numeration to reach Europe. Two well-known 13th century works on the new arithmetic are Sacrobosco's *Algorismus Vulgaris* and Leonardo of Pisa's *Liber Abbaci*. In the 13th and 14th centuries, the new arithmetic was taught in the European universities. A simple multiplication problem that can now be mastered in our kindergartens required the employment of a highly trained specialist. The solution of these simple arithmetic problems was held to be so wonderful that those who achieved it were regarded as tainted with magic! Numerous books on the new arithmetic were gradually being written and during the 16th century, the figure was around a thousand.

The introduction of a new set of numerals was not without resistance; in fact, the battle lasted four hundred years. It was not until 1600 that the Hindu-Arabic numeral system finally asserted its supremacy and was generally accepted as the standard system for computation. The widespread use of the new numerals was assisted by the availability of paper and the introduction of printing. In fact, it was printing which standardized the shapes of the numerals.

The invention of the numeral system has been described as one of the highest triumphs of inventive genius known in the history of scientific investigation. The difficulty of inventing such a system has prompted Laplace to remark that "it escaped the genius of Archimedes and of Apollonius of Perga, two of the greatest men of antiquity". The fact that it has been transmitted to all parts of the world and that it is still extremely useful today speaks for itself. When the system was generally accepted in Europe during the Renaissance, it laid the foundation for modern mathematics. As for its use and impact on other branches of science, commerce and the activities of daily life - I shall leave you to ponder on them.

May I end by drawing an analogy. We are at the beginning of a very exciting computer age. The advancement of computer science research has enabled even those who are illiterate in the subject to press buttons and obtain results. A time will come when computers will be as easy to operate as our numerals; they will become part and parcel of everyday living. By that time, how much of the tribulations of the computer scientists will be recorded and remembered? It is therefore no wonder that the long struggles encountered in the attainment of our numeral system have been uncertain and obscure.

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