MATHEMATICS EDUCATION IN INSTITUTE OF EDUCATION

Wong Hee Sing
Institute of Education

The Institute of Education is the only institute in Singapore responsible for teacher education, the pre-service training of primary and secondary school teachers and the in service training of qualified practising teachers, the preparation of university graduates for the Diploma in Education and the Master of Education and PhD in Education awarded by the National University of Singapore.

In contrast to the other tertiary institutions of Singapore where training for a variety of professions and occupations may be available, training at the Institute of Education is confined to training for the Education service. This is reflected in all the courses offered by the Institute.

The Mathematics Department in the School of Sciences shares the responsibility with other departments in the other schools of the Institute in the pre-service and in-service training of teachers of mathematics for primary, secondary and pre-university levels. It is not responsible for the Master of Education or Ph D in Education courses nor for the training of pre-primary level teachers. The former is the responsibility of the School of Advanced Studies and the latter the Department of Early Childhood Education.

While in the past, the Department has offered ad hoc in-service mathematics courses to upgrade the mathematics background of non-graduate mathematics teachers teaching modern mathematics at the secondary level, in the pre-service training programmes mathematics is not offered in isolation but in conjunction with the methodology of teaching mathematics courses.

Listed in the following are the different categories of pre-service and in-service teachers of mathematics with the corresponding preadmission mathematical background and the mathematics courses or courses with mathematics content prescribed under the respective course programmes. For the Concurrent training programmes, students perform teaching practice in schools and attend course lectures concurrently throughout their training period. In the Full-time training programmes students attend course lectures full time and are released full time for block teaching practice in schools.

Concurrent Training

Category	Mathematical background	Course in mathematics (Content) prescribed
Pre-Service:		
Dip Ed		
Those offering Major Special Method Mathematics (In preparation for teaching the subject at upper secondary and pre-university levels)	Degree or 3rd year university mathematics	None except those without Applied Mathematics or Statistics background. They are given a supplementary prerequisite 30 hour Introduction to Statistics course outside the overall requirements of their training programme
Those offering Minor Special Method Mathematics (In preparation for teaching the subject at secondary level)	'A' Level mathematics	None
Cert Ed ('A' level)		
Those offering Special Method Mathematics (In preparation for teaching the subject at upper primary and lower secondary levels)	'A' Level mathematics	Special Academic Subject Mathematics course of 120 hours
Cert Ed ('O' level General)		
Compulsory for all (In preparation for teaching at primary level)	'O' Level mathematics	60 hours of mathematics content out of 90 hours of Group A Teaching Subject Mathematics pedagogy course
Cert Ed ('O' level, Tamil)		
Those offering Group A Teaching Subject Mathematics (In preparation for teaching at primary level)	'O' Level mathematics	60 hours of mathematics content out of 90 hours of Group A Teaching Subject Mathematics pedagogy course (Same course as that for the Cert Ed ('O' level, General)

Category	Mathematical background	Course in mathematics (content) prescribed
Cert Ed ('O' level, PPLP) Compulsory for all (In preparation for teaching at Pre-Primary and Lower Primary levels)	Unspecified	60 hours of mathematics content out of 90 hours of Group A Teaching Subject Mathematics pedagogy course
In-Service: *Adv Cert Ed* Those offering Special Academic Subject Mathematics (Upgrading qualified non-graduate mathematics teachers teaching at secondary level)	'A' Level mathematics or previous and hoc In-Service course of Further Mathematics for secondary school mathematics teachers	120 hours of mathematics out of 135 hour course. 15 hours for methodology of teaching mathematics
Full-Time Training		
Pre-Service: *Dip Ed* Those offering Mathematics Special Method 1 (In preparation for teaching up to Pre-U level)	Degree or 3rd year university mathematics	None
Those offering Mathematics Special Method 2 (Essentially the same course as Special Method 1) (In preparation for teaching up to Sec 4 level)	'A' Level mathematics	None
Cert Ed ('A' level, General) Compulsory for all (In preparation for teaching at primary level)	Unspecified	Mathematics incorporated as part of 90 hour Teaching Subject (A) Mathematics pedagogy course

Category	Mathematical background	Course in Mathematics (content) prescribed
Cert Ed ('O' level, Tamil) Those offer Teaching Subject (Group B) (In preparation for teaching at primary level)	'O' Level mathematics	Mathematics incorporated as part of 60 hour Teaching Subject (B) Mathematics pedagogy course
Cert Ed ('O' level Early Childhood Education) Compulsory for all (In preparation for teaching at Pre-Primary and Lower Primary levels)	Unspecified	40 hours of mathematics out of a 120 hour Teaching Subject Group A Mathematics pedagogy course

The Concurrent training programmes are being phased out and replaced by the Full-time training programmes which were introduced in the July Semester of 1980.

Ideally, the pre-university level mathematics teacher should have a postgraduate honours degree in mathematics and the general certificate 'O' Level mathematics teacher a general degree mathematics background which would have included both Pure and Applied Mathematics or Statistics. Other topics that might be included in a desirable mathematics background for the Dip Ed student would be those that are related to the 'O' and 'A' level mathematics syllabi such as Mathematical Structure, for example.

Mathematics attained at level of about two years of undergraduate study should be sufficient for teaching the subject at the lower secondary level.

Students from the Concurrent Cert Ed ('A' level) programme were, in the main, those with only a background of traditional 'A' Level mathematics. As they were expected to teach up to secondary two level mathematics that includes a substantial proportion of modern topics, the Special Academic Mathematics course for them was designed to meet this special need.

Mathematics as part of the 90 hour Group A Teaching Subject for the Concurrent Cert Ed ('O' level, General) and as part of the 90 hour Teaching Subject A for the Full-time Cert Ed ('A' level, General) consist mainly of those topics in modern mathematics that are related to primary mathematics such as the number systems, numeration systems (number bases), relations and function. This is to provide the students with some idea of a higher level perspective of the primary mathematics which they would be teaching. It would not be undesirable if those teaching at the upper primary level have an additional year of mathematics at Pre-University One level.

In the recent past of 1980, because of a chronological doubling up of running both Concurrent and Full-time training programmes there have been classes of 60, 70 and

90 students for the Concurrent Cert Ed 'O' and 'A' level mathematics courses. For the January 1981 semester, however, class sizes of these courses for both Concurrent and Full-time training programmes have not exceeded 40 students. This was achieved through the distribution of the resultant excess teaching load among all members of the teaching staff of the Department.

Over the past few years the Institute has had difficulties in recruiting lecturing staff with the necessary academic and professional qualifications and experience to fill the vacancies in the Department, although offers have been made to a number of interested applicants who were suitably qualified.

Appendix 1

CERTIFICATE IN EDUCATION 'A' LEVEL (Concurrent Training)

Course Title: Specialist Academic (SA) Mathematics

Course Code: MA 13101

Duration: 120 h (2 credits)

Objective: To upgrade students knowledge of mathematics beyong the secondary school level.

To provide the necessary background knowledge of Modern Mathematics for the methodology course.

To enable students to teach the mathematics syllabus up to Secondary Two level.

Content:

1 **Set Theory**

Idea of a set. Element, subset and empty set Finite and infinite sets. Universal set and complement of a subset. Union and intersection of two sets. The formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Venn Diagrams and their applications. De-Morgan's laws.

2 **Relation and Function**

The idea of relation. Cartesian product of two sets. One to one relation, many to one relation, one to many relation, and many to many relation. Domain and range. Inverse relation. Reflexive, symmetric and transitive properties.

3 **Numeration System (Multibase)**

Addition, subtraction, multiplication, and division in any base. Conversion of number from one base to another.

4 **Number System**

Closure, associative, commutative and distributive laws. Identity and inverse.
Natural numbers, integers, rational numbers, irrational numbers, real numbers and complex numbers.

5 **Groups, Rings and Fields**

Definition of a group, Abelian group, subgroup, isomorphic groups. Ideas of a ring and field. Laws of operation in the algebra of sets.

6 Matrices

mxn matrix. 2x2 matrix. Matrix addition, subtraction and multiplication. Zero matrix, unit matrix and inverse matrix. The inverse and identity of the 3x3 matrix. Application to solution of simultaneous equations in more than two unknowns.

7 Determinants

Determinants and laws of manipulation.
Use of Crammer's rule to solve simultaneous equations in two and three unknowns.

8 Transformation Geometry

Geometry of Euclidean space based on the transformation of translation, rotation, reflection, enlargement. Use of 2x2 matrices to represent transformations. Column-vector notation $\begin{pmatrix} x \\ y \end{pmatrix}$ for points in the plane and pre-multiplication of column vector by 2x2 matrices. Ideas of shearing and stretching.

9 Inequality

Nature of inequality. Equivalent statements of inequality. Formulation and proof of "theorems" on operations with and combinations of inequalities.

10 Linear Programming

Graphing of $ax + by + c \geqslant 0$ and $ax + by + c \leqslant 0$.
Problems in linear programming and their graphical solutions.

11 Vectors

Ideas of vectors. Addition of vectors. Scalar product and vector product. Simple application e.g. resultant and relative velocity, geometry.

12 Topology

Intuitive topology.
Topological transformation. Euler's formula for simple polyhedron. Connectivity, genus. Euler characteristics of a surface. Surfaces, networks. Unicursal problem.

13 Logic

Proposition and connectives. Truth table. Propositional formulae. Boolean algebra. Boolean algebraic operations applied to logic.

14 Simple Probability

Permutation and combination. Addition and multiplication laws of probability. Probability distribution.

15 Statistics

Representation of data. Measures of central tendency: mean, median, mode. Quantiles: quartiles, deciles, percentiles. Dispersion: mean deviation, standard deviation, variance and quartile deviation.

Assessment: Examination at the end of the course. One paper of 3 hours.

Prerequisite: A pass in mathematics at GCE 'A' Level

CERTIFICATE IN EDUCATION 'O' LEVEL (GENERAL) (Concurrent Training)

Course Title: Teaching Subject Group A - Mathematics

Course Code: MAO8101

Duration: 90 h (1½ credits)

Objective: To provide students with a background knowledge of some fundamental ideas in mathematics.
To prepare students to teach mathematics at the primary level.

Course Description:

A. Content ("Written") Section (60 h)

1 Sets. Intersection and union of sets. Venn diagrams.
2 Natural numbers, whole numbers, integers, rational numbers, irrational numbers, real numbers. The number line.
3 Closure, commutative, associative and distributive properties. Identity and inverse.
4 2x2 matrix and its application to the solution of equations.
5 Multibase arithmetic.
6 Relations and functions. Domain, codomain and range. Graphs of relations and functions.
7 Inequalities. Solution of inequalities, graphs.
8 Geometrical transformations.
9 Vectors. Application of vectors to geometrical problems.
10 Groups.
11 Permutations and combinations. Introduction to probability.
12 Frequency distributions. Representation of data. Averages. Dispersion. Standard Deviation.

Assessment: Examination. A written paper of 1 h.

ADVANCED CERTIFICATE IN EDUCATION (In-Service Course)

Course Title: Mathematics Specialist Academic Subject MA 43101

Length of Course: 135 hours

Objective: (i) To upgrade the teachers' knowledge of mathematics content.
(ii) To enable the teachers to teach Mathematics Alt. C at GCE 'O' level.
(iii) To upgrade the teachers' knowledge of methodology in mathematics teaching.

Syllabus:

1 Set Theory

Idea of set. Elements, subsets, empty set, equality of sets, universal set, complement of a set, equivalent sets, Venn diagrams, number of elements in sets. Algebra of sets. De-Morgan's laws. Operations with sets: Union (U), intersection (∩) difference (-), symmetric difference (Δ).

2 Relation and Function

One-to-one correspondence, ordered pairs, Cartesian product, relations, equivalence relations. Idea of functions, representation by arrow diagrams, domain and range of a function. Existence of an inverse for a one-one mapping, composition of mappings. Simple algebraic

functions and their graphs.

3 Groups, Rings and Fields

Group and subgroup. Cyclic and modular groups; generators of a cyclic group.
Homomorphism; isomorphism; isomorphic groups and examples.
Examples of groups e.g. permutation groups, function groups, symmetry groups.
Some theorems in group theory. Lagrange's theorem. Cosets. Rings and examples of rings. Fields, finite and infinite fields.

Isomorphism between the field of complex numbers and the field of 2x2 matrices of form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ where a, b are real numbers.

4 Statistics

Measures of central tendency, measures of dispersion. Probability distributions. Sampling, estimation, test of hypotheses, error analysis. Regression and correlation, analysis of variance.

5 Matrices

Some types of matrices: triangular matrices, scalar matrices, diagonal matrices, symmetric matrices, skew-symmetric matrices, etc. Conjugate of matrices, Hermitian matrices, skew-Hermitian matrices, equal matrices, identity matrix, zero matrix, unit matrix, matrix addition, multiplication, properties of matrices. The adjoint of a square matrix. The inverse of 3x3 matrix. Application to simultaneous equations in two unknowns and three unknowns. Determinants and laws of multiplication. Sarrus rule for 3x3 determinants.

6 Transformation Geometry

Translation, rotation, reflection, enlargement, shearing and stretching, combinations of transformations.

7 Complex Numbers

The algebraic and trigonometric forms of complex numbers: real and imaginary parts, modulus and argument. Sum, product and quotient of two complex numbers. The Argand diagram. The triangle inequality $|a + b| \leqslant |a| + |b|$. Demoivre's theorem.

8 Inequality and linear programming

Simple inequality. Simultaneous linear equations and inequalities in not more than three unknowns. Applications of equalities and inequalities to simple problems, to linear relationships and graphs.
Inequalities and regions in the plane determined by inequalities of the form $ax + by + c \leqslant 0$ and $ax + by + c \geqslant 0$ Graphical and algebraic solutions to problems in linear programming. Simplex methods.

9 Vector Spaces

Vector spaces (over the real numbers), scalar product, and orthogonal basis. Linear transformations of vector spaces and their matrices, characteristic polynomial, characteristic values and vectors.

10 Elementary analysis

Functions, limits, continuity, differentiation, integration.
Trigonometric and hyperbolic functions and their inverses. Rolle's theorem and consequences, Taylor expansion. Functions of several variables, partial differentiation.

11 Differential equations

Ordinary differential equations of first order, linear equations with constant coefficients, applications.

12 Theory of numbers

Integers, divisibility, quotient and remainder. Primes and composites, the sieve of Eratosthenes, the number of primes infinite.
The notion of a module of numbers, the module $xa + yb$ when a and b are given integers and x and y are any integers. Highest common factor. Uniqueness of factorisation of a composite. Least common multiple. Euclid's algorithm for highest common factor. Solution of linear Diophantine equation $xa + yb = n$ (Knowledge of continued fractions will not be expected).

Assessment is by: (1) Continuous assessment on course work.

(2) Final examination at the end of the course. Two papers of 3 hours each.

Appendix 4

CERTIFICATE IN EDUCATION GENERAL ('A' LEVEL) (Full-Time Training)

Course Title: Teaching Subject A (Mathematics) MA 58111

Duration: 90 hours

General Aims: To give students an in-depth knowledge of mathematics in the areas related to those contained in the primary school mathematics syllabus.

To equip students with knowledge of the basic principles in and the current approaches to the teaching of mathematics at the primary level.

Course Objectives: The course aims to provide students with

(i) an understanding of how mathematical concepts are formed and learned by children,

(ii) the ability to conduct group teaching in mathematics and to use a variety of approaches,

(iii) the ability to construct and use appropriate teaching aids,

(iv) knowledge of some techniques of assessment in mathematics and their uses for diagnostic and prognostic purposes,

(v) a thorough understanding of the mathematical concepts of the topics contained in the primary school syllabus.

Course Description:

This section caters for the development of the student's understanding of mathematics in those areas relevant to the primary mathematics syllabus.

1 Set Theory

Idea of a set. Element, subset, universal set and complement. Union and intersection. Venn diagrams and their application. De Morgan's laws.

2 Relation and Function

The idea of mapping, relation and function. Domain, codomain and range. Reflexive, symmetric and transitive properties. Inverse relations and functions.

3 Number Bases

Scale of notation. Addition, subtraction, multiplication and division in any base. Conversion of numbers from one base to another.

4 Number System

Natural numbers, integers, rational and irrational numbers, real and complex numbers. The number line. Ideas of closure, associativity, commutativity and distributivity. Identity and inverse elements. Definition of a group and simple examples of groups.

5 Statistics

Representation of data. Frequency distribution. Measures of central location - mean, median and mode. Cumulative frequency, quartiles, deciles and percentiles. Dispersion - range, mean deviation and standard deviation. The normal distribution.

Assessment is based on course assignments.

Appendix 5

CERTIFICATE IN EDUCATION ('O' LEVEL) - TAMIL COURSE (Full-Time Training)

Course Title: Teaching Subject (Group B) - Mathematics

Course Code: MA 58112

Duration: 60 h (2 credits)

Objectives:

(i) To provide the students with the necessary knowledge of some fundamental ideas in mathematics.

(ii) To provide the students with some basic principles, approaches, methods and techniques in the teaching of mathematics in the primary level.

(iii) To provide the students with some experiences in the construction and use of teaching aids in the mathematics teaching.

(iv) To provide the students with the knowledge of some fundamental methods of assessments in mathematics and the use of their results in diagnosis and remediation purposes.

Course Description:

1 Sets, intersection and union of sets. Venn diagrams.

2 The number system. The number line. Closure, commutative, associative and distributive properties. Identity and inverses.

3 Numeration system.

4 Relations and functions. Domain, codomain and range. Graphs of relations and functions.

5 Geometrical transformation.

6 Frequency distribution. Representation of data. Averages, dispersions and standard deviation.

Assessment is based on course assignments.