MATHEMATICAL EDUCATION IN SCHOOLS AND UNIVERSITIES IN THE FEDERAL REPUBLIC OF GERMANY*

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Introduction

The Federal Republic of Germany is made up of *Länder*† with each having a State Government, which has a certain degree of autonomy in running state affairs like cultural and educational matters. The schools are not under the direct control of the Federal Government in Bonn and, unlike Singapore, there is no unified school system in West Germany. The individual State Government runs its own ministry of education based on guidelines laid down by the Federal Government. This has the advantage of speedier communication and better understanding between the authority and the people. On the other hand, there are problems such as that for a child of schooling age, when his father is transferred from one state to another by his employer. On the whole, the school system is quite flexible and there is opportunity for individual development. However, the higher education system seems to show a partiality towards the elite group.

Elementary School to Gymnasium

Children in West Germany enter elementary school at the age of six. A "check-point" based mainly on class performance awaits them after the completion of their first four schooling years. On the recommendation of their teachers and with the consent of their parents, the more competent children are sent to *Gymnasium* to receive a more academically inclined education, which prepares them eventually for tertiary education. The other children remain in elementary school until the age of fourteen and thereafter, they are channelled into vocational training, where apprentice ships in various disciplines are available. The level of *Gymnasium* is equivalent to that from secondary school to pre-university class or junior college in Singapore. The final examination at *Gymnasium* is called *Abitur*. Not everyone in *Gymnasium* is qualified to sit this final examination. The dropouts of the qualifying test, *Mittlere Reife*, normally at the age of sixteen, may seek employment or undertake vocational training. The average age of students sitting *Abitur* is nineteen, that is, after they have completed thirteen schooling years.

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† Apart from West Berlin which has an unique status, the Federal Republic of Germany consists of 10 federal states *(Bundesländer)*.

As far as the school mathematics curriculum is concerned, an axiomatic approach with more emphasis on structural aspects of mathematics and less emphasis on techniques has been adopted. This indeed follows the tradition of German mathematical thinking, which dates back to David Hilbert towards the turn of the last century. As an illustration, in the example of dividing 37 by 6, a school child may now present the solution in the form $37 = 6.6 + 1$ following the division algorithm, instead of the traditional solution $37 \div 6 = 6$ with remainder 1. In some schools, the construction of the real number system using Dedekind cuts is presented to fourteen years old students. Furthermore, concepts like continuity and differentiability can be expected from a student completing his *Gymnasium*. However, the introduction of set language like $\cup$, $\cap$ and Venn diagrams to children at too early an age has turned out to be a failure. Most states have now rejected this curriculum.

University

The selection for entrance into universities in West Germany is based essentially on the results of *Abitur*, the final examination at *Gymnasium*, which is conducted by the individual State Government. On the average, a student spends five to six years preparing for his final examination in the university. Two final examinations are offered : *Diplom* and *Staatsexamen*. There is no yearly examination to check the progress of a student. Only one *Vor-Diplom* or *Vor-Examen* examination is conducted prior to the final examination. Students are allowed to do this examination which is conducted orally, after they have collected sufficient *Scheine* (certificates awarded by the Department of Mathematics to students who have successfully participated in tutorials, written tests or seminars) in courses like analysis, linear algebra, topology and numerical analysis.

Diplom candidates in addition to submitting a thesis have to sit for an oral examination. In the oral examination they are examined on the following :

1. Pure Mathematics,
2. Applied Mathematics,
3. a major subject normally related to the candidate's thesis,
4. a minor subject such as physics, economics, chemistry, biology or philosophy.

Under (1) and (2) a candidate will be examined in topics like algebra, analysis, function theory, ordinary differential equations, numerical methods and probability.

Staatsexamen candidates offer a major and a minor subject for the final examination. Those who major in mathematics are required to submit an academic exercise and to sit for an oral examination. In general, the oral examination has three main objectives.

1. To test the understanding of problems and methods arising from basic concepts in algebra, analysis and topology. A candidate may offer, for example, three of the following subjects with at least one from (a), (b) and (c).
   a. Analysis,
   b. Geometry,

c. Algebra and Number Theory,
d. Information Theory and Applied Mathematics,
e. Stochastics,
f. Foundation of Mathematics, Mathematical Logic and History of Mathematics.

2. To test the knowledge of selected subjects from applied mathematics specified for the course.

3. To test in depth the major subject of the candidate. A candidate is also expected to have some knowledge of the historical and current developments of mathematics. Those who offer mathematics as a minor subject are only required to sit for an oral examination. The oral examination is less demanding and its objective is stated in (1) above.

Generally both *Diplom* and *Staatsexamen* candidates look for their own examiners who have to be approved by the Department. The syllabus for each of the subjects to be examined is laid down by the examiners themselves.

Oral examination is a special feature in German universities. While it is always a difficult task for the examiner to find suitable problems for the oral examination, this examination system enables the examiner to test in depth a candidate's knowledge of a given subject. Also, the examiner may help the candidate by changing or modifying the question during the examination, which is not possible in a written examination.

Apart from having to attend lectures and related tutorials, the students have to participate in at least one seminar in the case of *Staatsexamen* candidates and in at least two seminars in the case of *Diplom* candidates. In a seminar, original research topics are distributed among the participants. The students may work individually or in groups and they take turns to present their papers. In most cases, problems for the thesis or academic exercise arise from the seminars. The professor in charge would distribute the thesis problems according to the performance of the students at the seminar. The thesis for *Diplom* is expected to be quite substantial,normally including some new results, and should show the candidate's ability to do independent research. The requirement for the academic exercise is lower. The courses leading to *Staatsexamen* are meant for those who want to become school teachers. These courses therefore include some subjects from didactic mathematics.

Under this apparently flexible system, a student has the distinct advantage of having more time to develop himself. On the other hand, he needs strict self-discipline and proper guidance, otherwise, he may face the danger of not having sufficient credits to do his *Vor-Diplom* or *Vor-Examen* A reform is underway to restrict the number of years that a student is allowed to spend in a university, and whether this is for better or for worse is still to be seen.