THE ZERO SETS OF FUNCTIONS IN THE NEVANLINNA CLASS*

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1. Introduction

The theory of Hardy and Nevanlinna classes in the unit disc U has been studied exhaustively for many years and constitutes one of the most beautiful chapters of complex function theory. (See [8], [14]) Throughout the theory, zero sets play a fundamental role because of the factorization theorems. Recently, generalizations to several complex variables have been vigorously pursued, firstly to polydiscs U^n (see [20]) and then to balls B_n and strictly pseudoconvex domains in $\mathbb{C}^n$ (see [25], [28]). Although some important results have been obtained, many problems remain and researchers in this area are still gathering momentum. It turns out that the theory in several variables has some similarities with the old one but possesses many new features. In what follows, I shall try to present some of the progress made in the last few years, particularly concerning the zero sets of functions in the Nevanlinna class.

2. One-variable results

Some results of the classical theory are recalled here for motivation and for comparison.

Let U be the unit disc in the complex plane $\mathbb{C}$ and $0 < p < \infty$. Denote by H(U) the set of all holomorphic

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functions in U. Define

$$\|f\|_p = \sup_{0<r<1} \left\{ \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p d\theta \right\}^{\frac{1}{p}}$$

$$\|f\|_o = \sup_{0<r<1} \frac{1}{2\pi} \int_0^{2\pi} \log^+ |f(re^{i\theta})| d\theta$$

$$\|f\|_\infty = \sup_{|z|<1} |f(z)| .$$

The space N(U), $H^p(U)$ and $H^\infty(U)$ are defined as the set of all $f \in H(U)$ with $\|f\|_o < \infty$, $\|f\|_p < \infty$ and $\|f\|_\infty < \infty$ respectively. For $0 < p < q < \infty$, it follows easily from Jensen's or Hölder's inequality that

$$N(U) \supseteq H^p(U) \supseteq H^q(U) \supseteq H^\infty(U) .$$

The zero sets of holomorphic functions are characterised by the following theorem.

Theorem 2.1. Let Ω be an open subset of $\mathbb{C}$.

(a) If $f \in H(\Omega)$, $f \not\equiv 0$, then $Z(f) = f^{-1}(0)$ is a discrete subset of Ω, hence countable.

(b) (Weierstrass). Conversely, if E is a discrete subset of Ω, then there exists $f \in H(\Omega)$ such that $E = Z(f)$.

The zero sets of functions in N(U) are characterized by the following.

Theorem 2.2.

(a) If $f \in N(U)$, $f \not\equiv 0$, and $Z(f) = \{a_n\}_1^\infty$ listed according to multiplicities, then the Blaschke condition

(2.1) $\sum_{1}^{\infty} (1-|a_n|) < \infty$

holds.

(b) Conversely, if $\{a_n\}_1^{\infty}$ is a sequence in U satisfying (2.1), then there exists f ε N(U) (in fact in $H^{\infty}(U)$) such that $Z(f) = \{a_n\}_1^{\infty}$.
For example, we can choose f to be the Blaschke product

$$B(z) = \prod_{k=1}^{\infty} \frac{a_k - z}{1-\bar{a}_k z} \frac{|a_k|}{a_k}$$

if all $a_k \neq 0$.

3. Zero sets in several variables

Let Ω be an open set in $\mathbb{C}^n$. Denote points in $\mathbb{C}^n$ by $z = (z_1, \ldots, z_n)$. A function f in Ω is holomorphic if it is holomorphic in each variable separately. Let H(Ω) denote the set of all holomorphic function in Ω . For f ε H(Ω), the zero set Z(f) is given by Z(f) = {z ε Ω : f(z) = 0}.

A subset E of Ω is an analytic subvariety of Ω if for each a ε E, there exist a neighbourhood W of a and holomorphic functions $f_1, \ldots, f_k$ in W such that

$$E \cap W = \{z \in W : f_1(z) = \ldots = f_k(z) = 0\}.$$

The zero sets of holomorphic functions have the following properties. (See [9].)

(1) Z(f) is a pure (n-1)-dimensional analytic subvariety of Ω. Its regular points (i.e. where df ≠ 0) form an (n-1)-dimensional complex manifold; its singular points form a countable union of lower dimensional complex manifolds.

(2) (Cartan). Conversely, if the cohomology group $H^2(\Omega, Z) = 0$, and E is a pure (n-1)-dimensional

analytic subvariety of Ω , then there exists $f \in H(\Omega)$ such that $E = Z(f)$.

(3) The volume of $Z(f)$ can be defined as the volume of the set of its regular points. We use the k-dimensional Hausdorff measure H_k to denote the k-dimensional volume. Note that H_o is just the counting measure so that $H_o(A)$ is the number of elements of A.

(4) Zero multiplicity

Let $a \in \Omega$. If $f \equiv 0$, define $\mu(a) = \infty$. If $f \not\equiv 0$, then f can be expanded in the form

$$f(z) = f_k(z-a) + f_{k+1}(z-a) + \ldots$$

where each f_j is a homogeneous polynomial of degree j, and $f_k \not\equiv 0$. Define $\mu(a) = k$.

μ is called the zero multiplicity of f at a. It has the following properties ([3]).

(i) μ is an upper semicontinuous function in Ω .

(ii) μ is constant on each component of the set of regular points of $Z(f)$.

4. The generalized Blaschke condition

Let Ω denote the unit polydisc $U^n = \{z \in \mathbb{C}^n : |z_j| < 1, 1 \leq j \leq n\}$ or the unit ball $B_n = \{z \in \mathbb{C}^n : \|z\| < 1\}$. Let

$$\partial\Omega = T^n = \{z \in \mathbb{C}^n : |z_j| = 1, 1 \leq j \leq n\} \text{ if } \Omega = U^n$$

$$\partial\Omega = S^{2n-1} = \{z \in \mathbb{C}^n : \|z\| = 1\} \text{ if } \Omega = B_n .$$

Let m be the measure on $\partial\Omega$ induced by the Lebesgue measure in $\mathbb{C}^n$, normalized so that $m(\partial\Omega) = 1$.

Define

$$\|f\|_p = \sup_{0<r<1} \left\{ \int_{\partial\Omega} |f(r\omega)|^p dm(\omega) \right\}^{\frac{1}{p}}$$

$$\|f\|_o = \sup_{0<r<1} \int_{\partial\Omega} \log^+ |f(r\omega)| dm(\omega)$$

$$\|f\|_\infty = \sup_{z\varepsilon\Omega} |f(z)| .$$

The spaces $N(\Omega)$, $H^p(\Omega)$ and $H^\infty(\Omega)$ are defined as the set of all $f \varepsilon H(\Omega)$ such that $\|f\|_p < \infty$ and $\|f\|_\infty < \infty$ respectively. For $0 < p < q < \infty$, we have

$$N(\Omega) \supseteq H^p(\Omega) \supseteq H^q(\Omega) \supseteq H^\infty(\Omega) .$$

The Blaschke condition (2.1) can be generalized and we have the following.

Theorem 4.1. (Chee [4], 1970)

If $f \varepsilon N(\Omega)$, $f \not\equiv 0$, and μ is the zero multiplicity of f then

$$(4.1) \qquad \int_o^1 dr \int_{\bar{\Omega}(r)} \mu(z) dH_{2k-2}(z) < \infty ,$$

where $\Omega(r)$ is the polydisc or ball of radius r .

When $n = 1$, (4.1) is equivalent to (2.1). In 1974, Malliavin [16] proved the analogous result for strictly pseudoconvex domains in $\mathbb{C}^n$. (See §8 below).

5. Examples

Theorem 4.1 shows that the generalized Blaschke condition is a necessary condition for the zero sets of functions in $N(\Omega)$. We give some examples to show that it is not a sufficient condition for the zero sets of various subspaces of $N(\Omega)$.

Definition: A subset E of Ω is a determining set (D-set) for a family $\mathcal{F}$ of holomorphic functions in Ω if $f \in \mathcal{F}$, $f = 0$ on E implies $f \equiv 0$ in Ω.

Theorem 5.1. (Chee [6])

For $n \geq 2$, there exists a pure $(n-1)$-dimensional subvariety V of U^n such that V is a D-set for $N(U^n)$ and

$$\int_0^1 H_{2n-2}(V(r))dr < \infty$$

where $V(r) = V \cap U^n(r)$.

If $n \geq 3$, V can be chosen so that $H_{2n-2}(V) < \infty$. Specifically, for $n = 2$, choose $\{a_k\}$ such that $0 < a_k < 1$,

$$\Sigma a_k^{3/2} = \infty, \quad \Sigma a_k^{5/2} < \infty.$$

Put $r_k = 1-a_k$, $V_k = \left\{(z_1,z_2) \in U^2 : z_1+z_2 = 2r_k\right\}$, $V = \bigcup_1^\infty V_k$.

For $n = 3$, choose $\{a_k\}$ such that $0 < a_k < 1$,

$$\Sigma a_k^2 = \infty, \quad \Sigma a_k^3 < \infty$$

Put $r_k = 1-a_k$, $V_k = \left\{(z_1,z_2,z_3) \in U^3 : z_1+z_2+z_3 = 3r_k\right\}$, $V = \bigcup_1^\infty V_k$. The case $n > 3$ can be reduced to $n = 3$.

Theorem 5.2. (Chee [6])

For $n \geq 2$, there exists a pure $(n-1)$-dimensional analytic subvariety V of B_n such that V is a D-set for $H^p(B_n)$ for all $p > 0$, and

$$\int_0^1 H_2(V(r))dr < \infty.$$

If $n \geq 3$, V can be chosen so that $H_{2n-2}(V) < \infty$.

Specifically, for $n = 2$, choose α such that $\frac{1}{2} < \alpha < 1$. Let $r_k = 1 - k^{-\alpha}$,

$$V_k = \{z \in B_2 : z_2 = r_k\}, \quad V = \bigcup_1^\infty V_k .$$

For $n \geq 3$, choose α such that $\frac{1}{n-1} < \alpha < 1$. Let $r_k = 1-k^{-\alpha}$,

$$V_k = \{z \in B_n : z_n = r_k\}, \quad V = \bigcup_1^\infty V_k .$$

6. Comparison of zeros

For one variable, functions in $N(U)$, $H^p(U)$ and $H^\infty(U)$ have the same zero sets (characterized by the Blaschke condition). This is not true for several variables.

Theorem 6.1. (Rudin [20], 1968)

For all p, $0 < p < \infty$, there exists $f \in H^p(U^2)$, $f \not\equiv 0$ such that $Z(f)$ is a D-set for $H^\infty(U^2)$.

Theorem 6.2. (Miles [17], 1973)

For all $0 < p < q < \infty$, $n \geq 2$, there exists $f \in H^p(U^n)$, $f \not\equiv 0$ such that $Z(f)$ is a D-set for $H^q(U^n)$.

Theorem 6.3. (Rudin [21], 1976)

For all $0 < p < q < \infty$, $n \geq 2$, there exists $f \in H^p(B_n)$, $f \not\equiv 0$ such that $Z(f)$ is a D-set for $H^q(B_n)$.

7. Sufficient conditions

We now consider the problem: Given a pure $(n-1)$-dimensional analytic subvariety E of $\Omega = U^n$ or B_n, find sufficient conditions of a metric or geometric nature such that E is the zero set of an $N(\Omega)$-function or an H^p-function.

By Cartan's theorem (see §3(2) above), this is equivalent to the following: Given $f \in H(\Omega)$, $f \not\equiv 0$, under what conditions will there exist an $F \in N(\Omega)$ or $H^p(\Omega)$ such that $Z(F) = Z(f)$?

Until now, the problem has not been completly solved.

Some partial results are as follows.

Theorem 7.1. (Rudin [20], 1967)

If $f \in H(U^n)$ and $\mathrm{dist}(Z(f), T^n) > 0$, then there exists $F \in H^\infty(U^n)$ such that $Z(F) = Z(f)$ counting multiplicities.

Theorem 7.2. (Stout [26], 1968). Solution of Cousin II problem with bounded data.

Let $\{V_\alpha\}_{\alpha\in I}$ be an open cover of $\overline{U^n}$. If for all $\alpha \in I$, there exists $f_\alpha \in H^\infty(V_\alpha \cap U^n)$ such that for all α, β, $f_\alpha f_\beta^{-1}$ is invertible in $H^\infty(V_\alpha \cap V_\beta \cap U^n)$, then there exists $F \in H^\infty(U^n)$ such that for all α, Ff_α^{-1} is invertible in $H^\infty(V_\alpha \cap U^n)$.

By considering solution of Cousin II problem, Zarantonello obtained a zero set for Nevanlinna class.

Theorem 7.3. (Zarantonello [30], 1974)

Suppose $n \geq 2$, $f \in H(U^n)$, and suppose there exist $0 < r < 1$, and a continuous function $\lambda: [r,1) \to [r,1)$ such that

$$|z_n| < \lambda\left(\frac{|z_1|+\ldots+|z_{n-1}|}{n-1}\right), \tag{7.1}$$

for all $(z_1,\ldots,z_n) \in Z(f) \cap Q^n(r,1)$, where $Q(r,1) = \{z \in \mathbb{C}: r < |z| < 1\}$, then there exists $F \in N(U^n)$ such that $Z(F) = Z(f)$.

Using Theorem 7.1 and Theorem 7.2, we showed that F can in fact be chosen in $H^\infty(U^n)$.

Theorem 7.4. (Chee [5], 1976)

In Theorem 7.3, F can be chosen in $H^\infty(U^n)$. This generalizes Theorem 7.1.

8. Sufficient conditions in strictly pseudoconvex domains

The case of the ball B_n or more generally, strictly pseudoconvex domains in $\mathbb{C}^n$ has received much attention lately, culminating in the remarkable results of Skoda and Henkin characterizing the zero sets of functions in the Nevanlinna class.

Let Ω be a relatively compact open set in $\mathbb{C}^n$ with C^k boundary, $k \geq 2$. If there exist a neighbourhood W of $\partial\Omega$, a C^k map $\rho : W \to \mathbb{R}$ and a number $\gamma > 0$ satisfying

$$\text{grad } \rho \neq 0 \quad \text{on} \quad W,$$

$$\Omega \cap W = \{z \in W : \rho(z) < 0\}, \quad \text{and}$$

$$\sum_{i,j,k=1}^{n} \frac{\partial^2 \rho(z)}{\partial z_i \partial \bar{z}_j} w_i \bar{w}_j \geq \gamma |w|^2, \quad w \in \mathbb{C}^n, \quad z \in W,$$

then we say that Ω is a strictly pseudoconvex domain with C^k boundary. The unit ball B_n is a strictly pseudoconvex domain with C^∞ boundary; the unit polydisc U^n is pseudoconvex. Hardy and Nevanlinna classes can be defined in such domains as the set of those holomorphic functions for which

$$\sup_{r<0} \int_{\partial\Omega_r} |f|^p \, dm_r < \infty$$

$$\sup_{r<0} \int_{\partial\Omega_r} \log^+ |f| \, dm_r < \infty$$

where $\partial\Omega_r = \{z \in W : \rho(z) = r\}$ and m_r is the measure on $\partial\Omega_r$ induced by the Lebesgue measure on $\mathbb{C}^n$. (See [23], [24], [25], [28].)

In 1974, Malliavin proved that the generalized Blaschke condition is a necessary condition for zero sets of functions in $N(\Omega)$.

<u>Theorem 8.1.</u> (Malliavin [16], 1974)

Let Ω be a strictly pseudoconvex domain in $\mathbb{C}^n$ with C^3 boundary. If $f \in N(\Omega)$, $f \not\equiv 0$, then

$$(8.1) \qquad \int_{Z(f)} \delta(z)\mu(z)dH_{2n-2}(z) < \infty$$

where $\delta(z) = \mathrm{dist}(z,\partial\Omega)$, μ = zero multiplicity of f. When $\Omega = B_n$, (8.1) is equivalent to (4.1).

Zero sets with finite volume were considered by Laville [15] and Gruman [10]. Finally in 1975, Skoda and Henkin proved independently that (8.1) is a sufficient condition for zero sets of $N(\Omega)$.

Theorem 8.2. (Gruman [10], 1975)

Let Ω be a strictly pseudoconvex domain in $\mathbb{C}^n$ with C^3 boundary, $H^2(\Omega,Z) = 0$. If $f \in H(\Omega)$, $f \not\equiv 0$, $H_{2n-2}(Z(f)) < \infty$, then there exists $F \in N(\Omega)$ such that $Z(F) = Z(f)$.

Theorem 8.3. (Skoda [24], 1975; Henkin [12], 1975)

Let Ω be a strictly pseudoconvex domain in $\mathbb{C}^n$ with C^2 boundary, $H^2(\Omega,Z) = 0$, $H^1(\Omega,R) = 0$.

If $f \in H(\Omega)$, $f \not\equiv 0$, and $Z(f)$ satisfies (8.1), then there exists $F \in N(\Omega)$ such that $Z(F) = Z(f)$.

Combining with Theorem 8.1, we see that the generalized Blaschke condition completely characterizes the zero sets of function in $N(\Omega)$, if Ω is the ball B_n or its generalizations, the strictly pseudoconvex domains with $H^2(\Omega,Z) = 0$. However, no sufficient condition for zeros of $H^p(\Omega)$-functions have been found. For bounded functions, we have the following.

Theorem 8.4. (Range and Siu [19], 1973)

Solutions of Cousin II problem with bounded data.

Let Ω be a strictly pseudoconvex domain in $\mathbb{C}^n$ with C^2 boundary and $H^1(\Omega, \mathcal{O}^*) = 0$ where $\mathcal{O}^*$ is the sheaf of germs of non-zero holomorphic functions in Ω. Let $\{V_\alpha\}_{\alpha\varepsilon I}$ be an open cover of $\bar{\Omega}$. If for each $\alpha \varepsilon I$, there exists $f_\alpha \varepsilon H^\infty(V_\alpha \cap \Omega)$ such that for all α, β, $f_\alpha f_\beta^{-1}$ is invertible in $H^\infty(V_\alpha \cap V_\beta \cap \Omega)$, then there exists $F \varepsilon H^\infty(\Omega)$ such that

Ff_{α}^{-1} is invertible in $H^{\infty}(V_{\alpha} \cap \Omega)$.

When Ω is a strictly convex domain in $\mathbb{C}^n$, the above was proved by Stout [27] in 1973.

Using the above result, Taylor proved the following.

Theorem 8.5. (Taylor [29], 1975).

Let Ω be U^2 or B_2, $f \in H(\Omega)$, $f \not\equiv 0$. If V is an irreducible component of $Z(f)$, then there exists $F \in H^{\infty}(\Omega)$ such that $V = Z(F)$.

9. Applications

We give here some applications of the results on zero sets discussed above.

I. Interpolation

Let E be a pure (n-1)-dimensional analytic subvariety in U^n. A function $g : E \to \mathbb{C}$ is holomorphic if for all $z \in E$, there exist a neighbourhood W of z and a function G holomorphic in W such that $G = g$ on $E \cap W$. By Cartan's Theorem B, (see e.g. [20, Th. 7.12]), for all g holomorphic on E, there is a G holomorphic in U^n such that $G = g$ on E. We wish to consider the problem: If g is bounded on E, can its extension G be chosen to be bounded on U^n? A set E for which this is possible for all bounded g is called an interpolation set. For n = 1, interpolation sets are characterized by L. Carleson in 1958 as the set of all uniformly separated sequences in U. (See [8].) For several variables, we have the following partial results.

Theorem 9.1. (Alexander [1], 1969).

Let $n \geq 2$, E is pure (n-1)-dimensional analytic subvariety of U^n such that

$$\text{dist}(E, T^n) = r > 0, \tag{9.1}$$

$$
(9.2)\quad \begin{cases} \text{there exists } \delta > 0 \text{ such that if } 1 \le i \le n, \\ \alpha \neq \beta, \\ (z',\alpha,z'') \text{ and } (z',\beta,z'') \in E \cap [Q^{i-1}(r,1) \times U \\ \times Q^{n-i}(r,1)] \text{ then } |\alpha-\beta| \geq \delta. \end{cases}
$$

Then E is an interpolation set.

Using Theorem 7.4 and a result of Andreotti and Stoll [2], we obtained a generalization.

Theorem 9.2. (Chee [5], 1976)

Let $n \geqslant 2$, E a pure (n-1)-dimensional analytic subvariety of U^n which satisfies (7.1) and (9.2). Then E is an interpolation set.

For strictly pseudoconvex domains, Henkin has proved the following remarkable result.

Theorem 9.3. (Henkin [12], 1972)

If Ω is a strictly pseudoconvex domain in $\mathbb{C}^n$, and E is a submanifold (i.e. an analytic subvariety without singularities) of Ω in general position, then there exists a bounded linear operator $L : H^\infty(E) \to H^\infty(\Omega)$ such that $Lg = g$ on E.

II. Removable singularities

Suppose E is a closed subset of U^n, $f \in H(U^n-E)$. For $0 < p < \infty$, we say that $f \in H^p(U^n-E)$ if $|f|^p$ has an n-harmonic majorant in U^n-E. The problem is: For what E is it true that every $f \in H^p(U^n-E)$ extends to an $F \in H^p(U^n)$?

Theorem 9.4. (Parreau [18], 1951)

If $n = 1$ and E has logarithmic capacity zero, then every $f \in H^p(U^n-E)$ extends to $H^p(U^n)$.

Theorem 9.5. (Shiffman [22], 1968)

Suppose $n > 1$, $H_{2n-1}(E) = 0$. Then every $f \in H^\infty(U^n-E)$ extends to $H^\infty(U^n)$.

Theorem 9.6. (Cima [7], 1974)

Suppose $n > 1$, $1 \leq p \leq \infty$, E a pure (n-1)-dimensional analytic subvariety of U^n satisfying (9.1). Then every $f \in H^p(U^n - E)$ extends to $H^p(U^n)$.

Using Theorem 7.4, we obtained a generalization.

Theorem 9.7. (Chee [5], 1976)

Suppose $n > 1$, $0 > p > \infty$, E a pure (n-1)-dimensional analytic subvariety of U^n satisfying (7.1). Then every $f \in H^p(U^n - E)$ extends to $H^p(U^n)$.

Recently, Harvey and Polking [13] have obtained some interesting results on removable singularities.

10. Open problems.

We list here some problems which naturally arise out of the above discussion.

(1) Find sufficient conditions for the zero sets of functions $N(U^n)$, $H^p(U^n)$, $H^p(B_n)$ and $H^\infty(B_n)$.

(2) Is it true that if $f \in H^\infty(B_n)$, $n > 2$, then $Z(f)$ has finite volume?

(3) If $E = Z(f)$ for some $f \in H^\infty(U^n)$ and satisfies (9.2), is E an interpolation set?

(4) If $E = Z(F)$ for some $F \in H^p(U^n)$, and $f \in H^p(U^n - E)$, can f be extended to $H^p(U^n)$?

11. References

1 H. Alexander : Extending bounded holomorphic functions from certain subvarieties of a polydisc.
Pac. J. Math. 29(1969) 485-490.

2 A. Andreotti and W. Stoll : The extension of bounded holomorphic functions from hypersurfaces in a polycylinder.
Rice Univ. Studies 56 No.2 (1970) 199-222.

3 P.S. Chee : The Blaschke condition for bounded holomorphic functions.
Trans. Amer. Math. Soc. 148(1970) 249-263.

4 P.S. Chee : On the generalized Blaschke condition.
Trans. Amer. Math. Soc. 152(1970) 227-231.

5 P.S. Chee : Zero sets and extensions of bounded holomorphic functions in polydiscs.
Proc. Amer. Math. Soc. 60(1976) 109-115.

6 P.S. Chee : Determining sets for Hardy and Nevanlinna classes in several complex variables.
Preprint.

7 J. Cima : An extension theorem for H^p-functions.
Proc. Amer. Math. Soc. 42(1974) 529-532.

8 P.L. Duren : Theory of H^p spaces.
Academic Press, 1970.

9 Gunning and Rossi : Analytic functions of several complex variables.
Prentice Hall, 1965.

10 L. Gruman : The zeros of holomorphic in strictly pseudoconvex domains.
Trans. Amer. Math. Soc. 207(1975) 163-174.

11 G.M. Henkin : Continuation of bounded holomorphic functions from submanifolds in general position to strictly pseudoconvex domains.
Math. USSR. Izv. 6(1972) 536-563.

12 G.M. Henkin : Solutions with bounds for the equations of H. Lewy and Poincaré-Lelong. Construction

of functions of Nevanlinna class with given zeros in a strongly pseudoconvex domain.

Doklady Akad. Nauk SSSR, 224(1975) 771-774.

13 R. Harvey and J. Polking : Extending analytic objects.

Comm. Pure and Appl. Math. 28(1975) 701-727.

14 K. Hoffman : Banach spaces of analytic functions.

Prentice Hall, 1962.

15 G. Laville : Diviseurs et classe de Nevanlinna.

Thèse, Univ. Paris, June 1975.

16 P. Malliavin : Fonctions de Green dún ouvert strictement pseudoconvex et classe de Nevanlinna.

C.R. Acad. Sci. Paris 278(1974) A141-A144.

17 J. Miles : Zero sets in $H^p(U^n)$

Ill. J. Math. 17(1973) 458-464.

18 M. Parreau : Sur les moyennes des fonctions harmoniques et analytiques et la classification des surfaces de Riemann.

Ann. Inst. Fourier (Grenoble) 3(1951) 103-197.

19 M. Range and Y.T. Siu : Uniform estimates for the δ-equation on domains with piecewise smooth strictly pseudoconvex boundaries.

Math. Ann. 206(1973) 325-354.

20 W. Rudin : Function theory in polydiscs.

W.A. Benjamin, 1969.

21 W. Rudin : Zeros of holomorphic functions in balls.

Indag. Math. 38(1976) 57-65.

22 B. Shiffman : On the removal of singularities of analytic sets.
Mich. Math. J. 15(1968) 111-120.

23 H. Skoda : Zeros de fonctions de la classe de Nevanlinna dans les ouverts strictement pseudoconvexes.
C.R. Acad. Sci. Paris, 280(1975) A1677-1680.

24 H. Skoda : Valeurs au bord pour les solutions de l'operateur d"et characterization des zeros des fonctions de la classes de Nevanlinna.
Preprint.

25 E.M. Stein : Boundary behaviour of holomorphic functions of several complex variables.
Princeton University Press, 1972.

26 E.L. Stout : The second Cousin problem with bounded data.
Pac. J. Math. 26(1968) 379-387.

27 E.L. Stout : On the multiplicative Cousin problem with bounded data.
Ann. Scuola Normale Sup. Pisa, 27(1973) 1-17.

28 E. L. Stout : H^p-functions on strictly pseudoconvex domains.
Amer. J. Math. 98(1976) 821-852.

29 B.A. Taylor : Components of zero sets of analytic functions in $\mathbb{C}^2$ in the unit ball or polydisc.
Pac. J. Math. 59(1975) 253-265.

30 S.E. Zarantonello : The multiplicative Cousin problem and a zero set for the Nevanlinna class in the polydisc.
Trans. Amer. Math. Soc. 200(1974) 291-313.