ABSTRACTS OF SHORT TALKS

REPRESENTATION OF NUMBERS BY CASCADES

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A cascade C is defined as a sum of binomial coefficients

$$C = \binom{a_h}{h} + \binom{a_{h-1}}{h-1} + \ldots + \binom{a_t}{t}$$

where $a_h > a_{h-1} > \ldots > a_t$. In this expression, we assume that $\binom{a}{h} = 0$ whenever $a < h$. Given a cascade C and a sequence $\varepsilon = \langle \varepsilon_h, \varepsilon_{h-1}, \ldots, \varepsilon_t \rangle$ of signs (i.e. $\varepsilon_i = +1$ or -1 for each i), we define

$$\varepsilon C = \varepsilon_h \binom{a_h}{h} + \ldots + \varepsilon_t \binom{a_t}{t} .$$

Also, we put

$$\alpha C = \binom{a_h}{h+1} + \binom{a_{h-1}}{h} + \ldots + \binom{a_t}{t+1} .$$

We shall prove that for any sequence $\langle n_0, n_1, \ldots, n_s \rangle$ of integers, there exist a cascade C and a corresponding sequence ε of signs such that $n_i = \varepsilon \alpha^i C$ for $i = 0, 1, \ldots, s$ where $\alpha^0 C = C$, $\alpha^1 C = \alpha C$, $\alpha^2 C = \alpha(\alpha^1 C)$, and recursively, $\alpha^n C = \alpha(\alpha^{n-1} C)$.

This paper is written jointly by C. C. Chen and D. E. Daykin and published in the *Proceedings of the American Mathematical Society*, Volume 59, Number 2, September 1976.

NUMERICAL SOLUTION OF A TWO-POINT BOUNDARY VALUE PROBLEM USING SPLINES

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In the study of the interaction of radiation with temperature in interstellar medium, one obtains a pair of second order non-linear ordinary differential equations with conditions at the two end points. This two-point boundary value problem is converted into a minimization problem with linear constraints, using cubic splines and introducing an error function. The Goldfarb-Lapidus gradient projection algorithm is then used to solve the minimization problem. This method is an adaptation of Sirisena's technique for solving two point boundary-value problems *(J. Optimization Theory and Applications,* Vol. 16, 1975).

This work is done jointly with Prof. K. K. Sen and Dr. A. N. Poo.

CONVOLUTION FUNCTIONS ON COMPACT ABELIAN GROUPS

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Let G be an infinite compact abelian group with character group $\hat{G}$. For $r > 0$, define $A_r(G)$ to be the set of all $f \in L_1(G)$ such that the Fourier transform $\hat{f} \in \ell_r(\hat{G})$. For $r > 0$ and $s > 0$, let $A(r,s)(G)$ be the set of all $f \in L_1(G)$ with $\hat{f}$ belonging to the Lorentz space $\ell(r,s)(\hat{G})$.

THEOREM 1. *Let $1 < p \leq 2$, $1 < q \leq 2$ and let $1/r = 1/p + 1/q - 1$. Then $L_p(G) * L_q(G) \subset A_{r'}(G)$, $1/r + 1/r' = 1$, and equality holds if and only if $p = q = 2$.*

THEOREM 2. *Let p, q, r be as in Theorem 1 and let* $1/s = 1/p + 1/q$. *Then we have*

(i) *there exist* $f \in L_p(G)$, $h \in L_q(G)$ *such that* $f * h \notin A(\beta,\gamma)(G)$ *for all* $\beta < r'$ *and all* $\gamma > 0$; *and*

(ii) *if* $0 < s_o < s$, *then there exist* $f \in L_p(G)$, $h \in L_q(G)$ *such that* $f * h \notin A(r', s_o)(G)$.

A corollary of Theorem 2 shows that Young's inequality is the best possible in some sense. A result of R. L. Lipsman *(Duke Math. J. 36(1969), 765-780)* and a theorem of U. B. Tewari and A. K. Gupta *(Bull. Austral. Math. Soc. 9(1973), 73-82)* are immediate consequences of Theorem 2.

REPRESENTATIONS OF LINEAR NILPOTENT GROUPS OF CLASS TWO

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An irreducible complex representation of a nilpotent group G of class two is faithful if and only if it has degree $G : [Z(G)]^{\frac{1}{2}}$. A full account of the representations of such a group will be given.

A PRESENTATION OF A GROUP

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A presentation, comprising a generating set and a set of defining relations, is obtained for Γ the automorphism group of the free group of rank two. The group Γ is related to the group of non-singular transformations of a (Euclidean) vector space. The generators of Γ correspond to the elementary transformations of rotation, reflection and shear. The defining relations allow a given automorphism to be expressed as a product of a minimum number of shears. The standard form is not unique but it has further interesting properties.

ON A CONJECTURE OF ERVIN FRIED

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A directed, simple graph G of order $\geqslant 3$ is said to satisfy Fried's conditions if any two distinct vertices of G have a unique upper bound and a unique lower bound. Ervin Fried conjectured that if G is finite and G satisfies Fried's conditions, then G has a triangle. E. C. Milner proved that if G is finite and that G satisfies Fried's conditions such that G is triangle-free, then G is regular and hence by a result of H. H. Teh, G is a quasi-group graph. We now prove that there is no finite directed abelian-group graph G satisfying Fried's conditions such that G is triangle-free. We further prove that if G is a finite directed graph satisfying Fried's conditions and that G is triangle-free, then the order of G is $\geqslant 31$.

THE HYPERCENTRE OF A FINITE GROUP

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Let G be a group. Let

$$1 = Z_0(G) \leq Z_1(G) \leq Z_2(G) \leq \dots$$

be the upper central series of G (i.e., $Z_1(G)$ is the centre of G and $Z_{i+1}(G)/Z_i(G) = Z_1(G/Z_i(G))$ for $i \geq 0$). The terminal member of the above series is called the *hypercentre* of G. We report the following result on the hypercentre of a finite group.

THEOREM. *Let G be a finite group. A subgroup X of G lies in the hypercentre of G if and only if $X \cap S$ lies in the hypercentre of S for every soluble subgroup S of G.*

DESIGN OF DISTURBANCES ABSORBING CONTROLLERS FOR LINEAR STOCHASTIC SYSTEMS

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In this paper, techniques have been developed for rejecting, utilizing, or minimizing the effects of internal and external disturbances in linear control systems. The disturbances are assumed to have known waveform structure, such as random step functions, random ramp functions, random sinusoidal functions, or more complicated functions; but their instantaneous amplitudes, phases, etc. are random and unknown. No statistical information about the disturbances is required and/or used. The resulting control system is disturbance-adaptive in nature, so that it has marked improvement in its performance when compared with a control system designed under the conventional methods. The design techniques developed have been successfully applied to the design of various real world control systems, some of which are shown in this paper.

AN INTEGRAL EQUATION WITH A PROBABILISTIC APPLICATION

Louis H. Y. Chen

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Let v be a real-valued, bounded and Borel measurable function defined on $[0,\infty)$, μ be a real measure defined on the Borel subsets of $[0,\infty)$ but with no atom at zero, and λ be a positive number. Define $S_{\lambda,\mu} = e^{\lambda(\mu-\delta)}$ where δ is the Dirac measure at zero. It is proved that there exists a real-valued, Borel measurable function f defined on $[0,\infty)$, with $\sup_{w\geq 0} |wf(w)| < \infty$ and satisfying the integral equation

$$wf(w) - \lambda \int tf(w+t)d\mu(t) = v(w),$$

if and only if $\int v dS_{\lambda,\mu} = 0$. It is also proved that the solution f is unique except at zero. Furthermore, an explicit expression for f and an upper bound for $\sup_{w\geq 0}|wf(w)|$ are obtained. The integral equation is then used to obtain a necessary and sufficient condition for the distributions of the row sums of an infinitesimal, finitely dependent triangular array of non-negative random variables to converge in total variation to the real measure $S_{\lambda,\mu}$.

THE MINIMAL α-DEGREE PROBLEM

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For each admissible ordinal α, a set $T \subset \alpha$ is of minimal α-degree if every set of strictly lower α-degree is α-recursive. One of the most important areas of study in higher recursion theory (cf. Sacks [3]) is that of minimal degrees in models of weak and strong set theories. The study is important as on the one hand it leads to the theory of imbeddings of lattices into degrees - which in turn yields undecidability results for various theories - and on the other hand it leads to the discovery by Sacks of the technique of forcing with perfect closed sets, a useful tool in studying independence results in set theory. The first theorem on minimal α-degrees was proved by Spector [6] on $\alpha=\omega$, refined by Sacks [4], and lifted to countable admissible ordinals by MacIntyre [2]. Shore [5] proved the existence of minimal α-degrees for all Σ_2-admissible ordinals α, and recently Maass [1] improved this result to the case $\sigma 2p(\alpha) \leq \sigma 2cf(\alpha)$. For various reasons, the remaining case stays insurmountable. In this talk we state a characterization theorem (via the notion of genericity) which we believe implies that minimal α-degrees do not exist for some α's.

THEOREM. Assume that α is not Σ_2-admissible. A regular, hyperregular set T is of minimal α-degree if and only if it is generic.

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THE FUNDAMENTAL GROUPOID OF A SPACE ASSOCIATED WITH A COVER

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Let U be any cover of a topological space X. We construct a classifying space BU associated with the cover U. This space has been considered by G. Segal who has proved that the projection $p : BU \to X$ is a homotopy equivalence if U is numerable. This of course implies that $\pi p : \pi BU \to \pi X$ is an equivalence of fundamental groupoids if U is numerable. We have proved that p induces an equivalence of fundamental groupoids when the interiors of the elements of U cover X and have showed how this is related to a theorem of Macbeath-Swan relating $\pi_1(G)$ to G in the case when group G acts on X.

This is part of the author's Ph.D thesis submitted to the University of Wales in December 1976.

THE LINEAR AUTOMORPHISMS OF CERTAIN IRREDUCIBLE LINEAR GROUPS

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Let G be a subgroup of the general linear group GL(r,F) over some field F which is splitting for G. The group of linear automorphisms of G is defined by

$$\text{lin aut } G = \{\phi \in \text{aut } G : \text{there exists } y \in GL(r,F) \text{ such that } x\phi = y^{-1}xy \text{ for all } x \in G\},$$

and the group of inner automorphisms of G is defined by inn G = {ϕ ε aut G : there exists y ε G such that $x\phi = y^{-1}xy$ for all x ε G}.

We study the structure of lin aut G/ inn G for certain finite q-groups of class 2 with cyclic centre, where q is an odd prime, and extend some results of David L. Winter on the automorphism group of an extraspecial p-group.

A CHARACTERISATION OF FINITE SIMPLE GROUPS WITH A COMPONENT ISOMORPHIC TO $PSL_3(4)$

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It is well-known that the finite simple groups constitute the fundamental building blocks of all the finite groups. In order to understand groups better, one would naturally like to have a complete list of all finite simple groups. Hitherto, almost every finite simple group appears in one of the following 3 infinite families:

(1) cyclic groups of prime order,
(2) alternating groups A_n of degree n with $n \geq 5$,
(3) groups of Lie type.

Besides these, there are so far 26 exceptions- they are the so-called sporadic simple groups. Whether there are finitely many sporadic simple groups or whether they belong in one way or another to some infinite families remains an open question. Through the characterisation of simple groups one attempts to settle this problem.

The general characterisation problem is as follows: Let (X) be a given group property. Determine all finite non-abelian simple groups which satisfy property (X).

Recently a systematic approach towards this problem has been developed and we are led to a study of simple groups in two categories:

(1) groups of component-type,

(2) groups of non component-type.

A characterisation of groups of component-type by the existence of certain "standard subgroups" was introduced by Aschbacher.

In this connexion, I have proved the following:

THEOREM. Let G be a finite simple non-abelian group which possesses a standard subgroup A such that $A|Z(A)$ is isomorphic to $PSL_3(4)$. Assume that $2^{10}T|G|$. Then G is isomorphic to He or O'N.

MULTIPLIERS FROM L_1 TO A BANACH L_1-MODULE

Quek Tong Seng and Leonard Y. H. Yap

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Let G be a locally compact abelian group with Haar measure λ. Let L_p, $1 \leqslant p \leqslant \infty$, denote the usual Lebesgue space with respect to λ. Let $A \subset L_p$, $1 \leqslant p < \infty$, be a Banach L_1-module such that $\| \ \|_p < \| \ \|_A$, and let $\{e_\alpha\}$ be a net in L_1 such that $\| e_\alpha \|_1 = 1$ for all α and $\{e_\alpha\}$ is a common approximate identity for L_1, L_p and A. Define the *relative completion* $\tilde{A}$ of A to be the space

$$\tilde{A} = \{f \in L_p : \sup_\alpha \| f * e_\alpha \|_A < \infty \}$$

with the norm $\| f \|_{\tilde{A}} = \sup \| f * e_\alpha \|_A$.

THEOREM. *(L_1,A), the space of multipliers from L_1 to A, is isometrically isomorphic to $\tilde{A}$ if $p > 1$, or if $p = 1$ and $(L_1,A) \subset L_1$.*

This theorem and some of its consequences will be discussed.

A CONVOLUTION ALGEBRA OF OPERATORS

Stephen T. L. Choy

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Let S be a compact semigroup and let A be a unital Banach algebra. Denote C(S,A) the space of continuous functions from S to A with uniform norm. Let T be a bounded linear operator from C(S,A) to A. It is shown in this paper that the set W(S,A) of all weakly compact operators T which are multipliers of A can be formed as a Banach algebra with convolution operation. Furthermore if K(S,A) is the set of compact operators T which are multipliers of A, then the maximal ideal space $\Delta K(S,A)$ is $\Delta(C'(S)) \times \Delta(M(A))$, the Cartesian product of the maximal ideal spaces of the measure algebra C'(S) and the algebra M(A) of the multipliers of A.

A TABULAR ALGORITHM FOR FINDING THE PARTIAL QUOTIENTS OF CONTINUED FRACTIONS EQUIVALENT TO $y = \sqrt{N}$

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Research on continued fractions has revealed many interesting facets of the irrational number $y = \sqrt{N}$ where N is a rational positive integer. For example:-

(1) $y = \sqrt{N} = \sqrt{M^2+2M-1} = [M,\overline{1,(M-1)1,2M}]$, where M > 1.

Thus, $y = \sqrt{N} = \sqrt{34} = \sqrt{5^2+2\times5-1} = [5,\overline{1,4,1,10}]$ for M = 5.

(2) $y = \sqrt{N} = \sqrt{(2M+1)^2+(3M+2)} = [(2M+1),\overline{1,2,1,2(2M+1)}]$, where $M \geqslant 1$. Thus, $y = \sqrt{95} = \sqrt{(2\times4+1)^2+(3\times4+1)} = [9,\overline{1,2,1,18}]$, where M = 4.

(3) $y = \sqrt{N} = \sqrt{(6M+3)^2+12} = [(6M+3),\overline{M,1,1,(3M+1)(4M+2),(3M+1),1,1,M,2(6M+3)}]$ where $M \geqslant 1$. Thus, $y = \sqrt{237} = \sqrt{(6\times2+3)^2+12} = [15,\overline{2,1,1,7,10,7,1,1,2,30}]$ for M = 2.

There are literally thousands of these unique relations that can only be found by constructing a table of partial fractions for $y = \sqrt{N}$ and comparing the behaviour of particular groups of partial quotients that appear to form a pattern - by the analytic method. A synthetic approach is difficult if not impossible.

The tabular method presented is a simplified arithmetical algorithm that dispenses with the arduous and time-consuming algebraic algorithm that has to be written down and evaluated, term by term, in the conventional method of finding the partial quotients of $y = \sqrt{N}$.

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