NUMERICAL PUZZLES: COMPUTER-ASSISTED SOLUTION

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Many people (mathematicians included) derive great satisfaction from the solving of mathematical puzzles. School teachers who have the pleasure of teaching mathematics often use puzzles to stimulate and maintain interest. Of the many kinds of mathematical puzzles those of a numerical nature appeal to readers of a wide age range because of the directness and simplicity of information. Two examples of such numerical puzzles are:

1. ABCD Puzzle

   Find the four digits A,B,C and D such that $A^{B}C^{D}$ = ABCD

2. DIGITS Puzzle

   Arrange the nine digits 1,2,3,...,9 into three 3-digit numbers such that the second 3-digit number is twice as big as the first 3-digit number and the third 3-digit number is three times as big as the first 3-digit number.

Unfortunately, it is not easy to persevere with these puzzles till a solution is found. One reason is that an algebraic approach leads to nowhere (for the writer at least, but don't let this discourage you). Another reason is that brute force takes too much time — brute force meaning the consideration of all possibilities one by one. For example, in the DIGITS puzzle the possible values for the first 3-digit number 123, 124, 125, 126, etc., can be tested till a solution is found. (Pure mathematicians may, frown on such a primitive method).

An interesting compromise is to apply some reasoning to rule out certain possibilities and then let the computer handle the laborious process of testing the remaining possibilities. In the DIGITS problem the restrictions on the three 3-digit numbers are such that

1. no two digits are the same (1,2,3,...9 occur once and only

once among the three numbers)

2. the sum of the digits is 45 (1+2+3+...+9=45)
3. the first 3-digit number lies in the range from 123 to 329 and it cannot end with a 5 since any multiple of 5 ends with either a 5 or 0.

With these conditions in mind the computer program was written. The computer took only 7 seconds to come up with the correct solution:

192 384 576 .....(1)

As can be imagined the writer (not being a pure mathematician) was rather pleased with the results of this method. Imagine his astonishment when the computer printed out another three solutions!

219	438	657	.....(2)
273	546	819	.....(3)
327	654	981	.....(4)

Of course, the second solution is just a rearrangement of the first one but did you notice that before or after seeing this? The intriguing question, then, is: Do other numerical puzzles have more solutions than stated?

Of course this kind of activity assumes you have access to computer facilities. If you don't, you can always think of the procedure that you want to use and get a friend with computer programming experience to write and run the program. Readers who wish to pursue the matter further will find many numerical puzzles in the books listed below. The writer must confess that he did not pursue what he preached and so he would be delighted if readers should find and share any discoveries made.

ABCD Program

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10   REM PROBLEM: FIND INTEGERS A,B,C,D SUCH THAT A**B*C**D=ABCD
20   FOR I=1111 TO 9999
30   LET A=INT(I/1000)
40   LET B=INT(I/100)-10*A
50   LET C=INT(I/10)-100*A-10*B
60   LET D=I-1000*A-100*B-10*C
70   IF A**B*C**D=I THEN 600
500  NEXT I
600  PRINT "NUMBER SUCH THAT A**B * C**D = ABCD IS";I
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COMPUTER PRINTOUT

NUMBER SUCH THAT A**B * C**D = ABCD IS 2592

.....

DIGITS Program

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10   DIM A(10)
20   FOR I=123 TO 329
25   IF INT(I/5)=I/5 THEN 190
30   LET A(1)=INT(I/100)
40   LET A(2)=INT(I/10)-10*A(1)
50   LET A(3)=I-100*A(1)-10*A(2)
60   LET A(4)=INT(2*I/100)
70   LET A(5)=INT(2*I/10)-10*A(4)
80   LET A(6)=2*I-100*A(4)-10*A(5)
90   LET A(7)=INT(3*I/100)
100  LET A(8)=INT(3*I 10)-10*A(7)
110  LET A(9)=3*I-100*A(7)-10*A(8)
115  IF (A(1)+A(2)+A(3)+A(4)+A(5)+A(6)+A(7)+A(8)+A(9))=45 THEN 190
120  FOR J=1 TO 8
140  FOR K=J+1 TO 9
150  IF A(J)=A(K) THEN 190
160  NEXT K
170  NEXT J
180  PRINT I;2*I;3*I
190  NEXT I
200  END
```

COMPUTER PRINTOUT

```
192   384   576
219   438   657
273   546   819
327   654   981
```

Computer model: Facom

Language: Basic

References

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