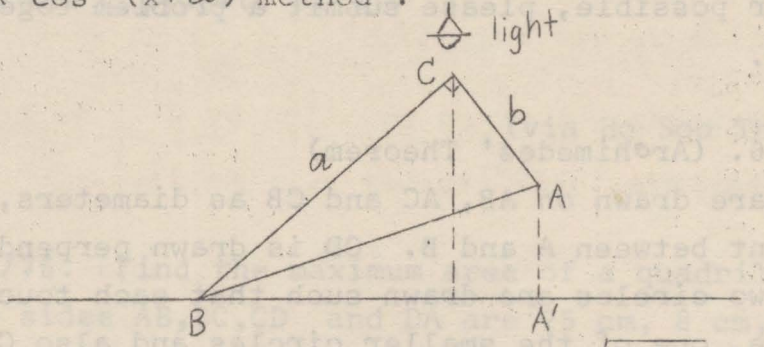
TEACHING NOTES

On finding the maximum of

$$f(x) = a \cos x + b \sin x$$

Chan Sing Chun

Functions of the form $f(x) = a \cos x + b \sin x$ occur frequently in pure and applied mathematics. Very often in school work we have to find the maximum value of $f(x)$. The following method used by me for teaching slow learners seem to be more effective that the usual method by differntiation or by the "R cos $(x+\alpha)$ method".



Consider 3 rigid rods BC=a, CA=b, AB= $\sqrt{a^2+b^2}$ forming a right angled $\triangle$ at C. The system is pivoted freely at B on a table and $\triangle$ABC is in a vertical plane, as shown in the figure. A light is placed above $\triangle$ ABC , and $\triangle$ ABC is rotated anticlockwise about B with BA initially on the table. It may be seen that the shadow cast by the triangle is BA' which is of length a cos x + b sin x where x is the angle which CB makes with the horizontal table. The shadow is longest when BA is on the table and its length is then $\sqrt{(a^2 + b^2)}$ and

$x = \widehat{ABC} = \tan^{-1}(\frac{b}{a})$. Hence the maximum value of a cos x + b sin x is $\sqrt{(a^2 + b^2)}$ and $x = \tan^{-1}(\frac{b}{a})$.

It is then not difficult to convince the pupils that the least value is $-\sqrt{(a^2 + b^2)}$. I would like to know from teachers what visual aids could be used for the minimum value. Negative shadow?