# 5. Norbert Wiener (1894 - 1964)

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To the general public, Norbert Wiener is known as the founder of the science of cybernetics (the theory of control and communication in the animal and the machine). Perhaps less well-known to the laymen is the fact that Wiener is a mathematician of the first rank who contributed towards the advancement of electrical engineering, physics and biophysics. In doing so, he was upholding the tradition of the great mathematical universalists of the past three centuries. His mathematics had often been inspired by physical problems and his mathematical thinking was guided by a deep physical intuition. Yet his mathematical works are among the purest of pure mathematics. So pure that G. H. Hardy [1], the British mathematician and the archetypal protagonist of "art for art's sake", thought that Wiener's "engineering-inspired" claims were, if not sheer humbug, mere professional politicking. Wiener was also a philosopher interested in the implications of the new automated technology, and an unusual autobiographical writer who has given us a frank and detailed account of the emotional and intellectual vicissitudes of his life and career in his two books *Ex-prodigy* [2] and *I am a mathematician* [3].

In the shadow of brilliance. Norbert Wiener was born the eldest son of Leo Wiener and Bertha Kahn Wiener on 26 November 1894 in Columbia, Missouri. His father was a Russian Jew who migrated to the United States as a penniless young man of eighteen after an unpromising start in Europe. His intelligence and his talent for languages soon turned him from a peddler into a foreign language teacher at the University of Missouri.

A self-made man with no formal university education, he nevertheless became a distinguished philologist and a Harvard University professor of Slavic languages who translated into English twenty-four volumes by the Russian writer Leo Tolstoy [4]. The mother Bertha Kahn Wiener was the daughter of a Jewish immigrant from Germany and had a non-Jewish grandparent.

Born into an intellectual tradition that could be traced back to a Grand Rabbi and even supposedly to the philosopher Moses Maimonides [5], Norbert Wiener soon felt the psychological pressures of the intellectual training and disciplines imposed by an exacting father who would accept nothing short of perfection. These demands instilled in young Norbert feelings of awe and humiliation and an emotional dependence on the family. He was an extremely sensitive person, and his adolescence was marked by a social ineptitude and insensitivity that was compounded by an agonising awareness of his Jewish background, the overt and subtle racial prejudices of that time and the need to achieve (and invariably intertwined, the fear of failure) that is often compulsively felt by a prodigy.

Wiener's precocity was manifested when he began reading at an early age. By the age of six, he was an avid reader of the books in his father's collection and was fascinated by zoology, botany, physics and chemistry in addition to the usual tales of travel and adventure. His omnivorous appetite for books was so great that by the age of eight his eyesight had deteriorated badly. His childhood ambition was to become a naturalist. Surprisingly, his precocity in mathematics was far beyond his poor manipulative skills in arithem tic. His father soon discovered that the ordinary school could not resolve this unusual situation of his precocious son. So up to the age of ten, young Wiener's education was directed by the father who worked out a programme on Mathematics (algebra and geometry) and languages (Latin and German).

The teaching by the father was supplemented by that of two tutors, one in chemistry and the other in Latin and German. Still, he had his full share of play with the neighbourhood children during this period of his childhood.

His three years, (1903 - 1906) at Ayer High School in Harvard, Massachusetts, brought him fond memories in his later years. The next three years (1906 -1909) saw him working for his bachelor's degree at Tufts College. His successful graduation left him in a state of physical exhaustion together with a feeling of bitterness at not being elected to the honour society Phi Beta Kappa. This bitterness affected him so deeply that he was to reject many honours belatedly bestowed upon him.

In 1909, barely fifteen, he enrolled in Harvard University to do graduate work in zoology. However, his physical slowness set against his intellectual impatience made him a disaster in the laboratory. With his dream to become a biologist shattered, he went to Cornell University the following year after having won a scholarship. His father had urged him to switch to philosophy, but the academic year at Cornell saw him emotionally disoriented and intellectually lost. His scholarship was not renewed and he returned to Harvard in 1911 to work for a doctorate in philosophy. He wrote his Ph.D. thesis on mathematical logic when he was hardly nineteen.

Up to this time, his mathematical education had been meagre. At Tufts, the mathematics courses were designed for engineering students, except for a special reading course, which was beyond him, in the theory of equations. At Cornell, he took but did not understand a course in the theory of functions of a complex variable. At Harvard, he studied axiomatic systems under the American mathematician, E. V. Huntington [6], and his thesis was written under the supervision of Karl Schmidt of Tufts College (he had been prevented from working with the philosopher Josiah Royce [7] by the latter's illness). Nevertheless, in the summer of 1913 prior to his departure for Cambridge University on a travelling fellowship, Wiener read, at Huntington's suggestion, *Modern algebra* by Maxime Bôcher [8] and *Projective geometry* by Oswald Veblen [9] and J. W. Young [10] .

From philosophy to mathematics. The Wiener family (complete with two sons and two daughters) planned to spend the winter of 1913 in Europe. However, Wiener would go to Cambridge University to study with Bertrand Russell [11], part of whose work on logic had been dealt with in Wiener's Ph.D. thesis. His short stay at Cambridge enlarged his perspectives in mathematics and physics, and he settled comfortably into the new environment of the individualistic and eccentric Cambridge dons.

Though essentially a logician, Russell had deep insight into the relevance of mathematics and physics to philosophy. At his suggestion, Wiener took courses by H. F. Baker [12] , Hardy, J. E. Littlewood [13] and J. Mercer [14]. Hardy impressed the young logician by his clarity, interest and intellectual power, and his course on analysis (particularly the Lebesgue integral) had a strong influence on Wiener's mathematical development. Wiener was also introduced to the three revolutionary papers of Albert Einstein [15], on relativity, the photoelectric effect and Brownian motion, and to the work of Niels Bohr [16] on the atomic nucleus. (Hardly did the young Wiener suspect that he would make a profound contribution to the theory of Brownian motion ten years later). All these were in addition to the courses given by Russell. During this time, Wiener had sensed the impossibility of building a complete and consistent logical system. This was borne out by Kurt Gödel [17] who showed years later that the consistency of the arithemetical system implies its own incompleteness (in the sense that there are arithemetical statements which cannot be proved or disproved within the system).

The May term (1914) saw some changes in Wiener's plan. Russell would be away in Harvard; so Wiener decided to spend the rest of the academic year at Göttingen University which had become the beehive of mathematical and scientific activities. At Göttingen, he studied mathematics with Edmund Landau [18] and David Hilbert [19], and philosophy with Edmund Husserl [20]. He benefitted little from the formal courses but the many contacts with various kinds of people helped him in his social adjustment towards other people. Above all, the meetings of the Mathematical Society taught him that "mathematics was not only a subject to be done in the study but one to be discussed with and lived with." [2].

On his return to the United States, he was again awarded a travelling fellowship by Harvard. He again chose Cambridge University and arrived amidst the gathering storms of the First World War. The once congenial atmosphere was now highly charged with a heavy gloom and a demoralizing bleakness. Shortly after, he returned to the United States.

Until the end of the war, he did not obtain any proper job in mathematics. A possible opening in the philosophy department at Harvard was closed to him after it was discovered that a janitor had leaked out to him the actual grades of his examination results when he was a student. He was at Columbia University for a short period and was an assistant in philosophy at Harvard where he also gave a free series of Docent Lectures [21] on logic. His impressions and experiences at Harvard did not endear that academic institution or its academicians to him. On his father's advice, he switched to a career in mathematics and obtained an instructorship at the University of Maine —— but this experience turned out to be a nightmare for him. He left teaching and made some unsuccessful attempts at enlisting for military service. Subsequently, he worked for a brief period as an apprentice engineer and then as a hack writer for *Encyclopedia Americana* .Finally, he helped in the computation of ballistic tables at the Aberdeen Proving Ground in Maryland. Though the work was not mathematically significant, he found stimulating the intellectual life with the other scientists and mathematicians working on the same project. With the signing of the armistice, he was discharged from the army in which he managed to get enlisted as a private in the last days of the war. Next came a short and abortive stint at reporting for the *Boston Herald*.

When the war was raging in Europe and on the high seas, Wiener continued his research work in logic although he did try his hand at some of the most difficult problems in mathematics such as the four-colour problem, Fermat's last theorem and the Riemann hypothesis. There was an attempt in 1915 to set up the axiomatic foundations of what is now known as topology. He found his results unsatisfactory and abandoned this field of research. He was comparatively a latecomer to modern mathematics. His real understanding of modern mathematics was first acquired by a chance reading of the mathematics books given to his sister Constance by the parents of her fiance G. M. Green when Green, a budding Harvard mathematician, succumbed to the post-war influenza epidemic. Among those books were *Théorie des equations integrales* by Vito Volterra [22], *Funktionentheorie* by W. F. Osgood [23], Lebesgues' [24] book on the theory of integration and Fréchet's [25] book on the theory of functionals. The final break in Wiener's hitherto uncertain career came in the spring of 1919 through the food offices of Osgood who secured for him a position in the mathematics department of the Massachusetts Institute of Technology (M.I.T.).

<u>Growth and fruition</u>. During the early twenties, the mathematics department of M.I. T. was relatively unknown. It was mainly a service department devoted to the teaching of calculus to engineering students. The younger members of the department were enthusiastically engaged in research, but it had no high standing in research. This environment proved to be a godsend to the young Wiener who often felt insecure about his own abilities and suspicious of the higher echelon of the academic hierarchy. He managed with a heavy teaching load of twenty

hours a week and got along well with his students, his eccentricities notwithstanding. He discussed freely with his colleagues and his self doubt was allayed by their encouragement.

Fréchet's book had inspired Wiener to ask the young mathematician I.A.Barnett [26] for a good research problem in functional analysis. The latter suggested "the problem of integration in function space". This "completely influenced the whole course of Wiener's work and his greatest achievements all stemmed from this problem." [27] Wiener also consulted O.D.Kellog [28] of Harvard on potential theory and solved significant problems in that field. Unfortunately, two doctorate students of Kellog were attacking the same problems but making less progress than Wiener. And when Wiener was asked to hold back the publication of his results so as not to affect the theses of those students he was very upset and unhappy. Anyway, N. Levinson [29] tells us that this matter was satisfactorily settled. [28].

At the request of his colleagues in the electirical engineering department, Wiener undertook to lay the mathematical foundations of the formal calculus of communication engineering developed by Oliver Heaviside [30] some twenty years before. His efforts culminated in his formulation of the theory of generalized harmonic anlaysis. A problem in this field led him to a still greater discovery of general results in the so-called Tauberian theorems [31].

All these achievements were made within the first decade of Wiener's career at M.I.T., and he had already established a world-wide reputation when he was in his early thirties. But he felt and for a long time remembered that it was not his fellow Americans who gave him recognition for his early successes. His rise in the academic ranks of MIT was slow. He became an assistant professor in 1924, an associate professor in 1929 and finally a professor in 1932.

The city of Strasbourg had just been re-Gallicized after the trauma of the First World War, and was selected as the venue of the International Mathematical Congress [32] of 1920. However, as a punishment, the Germans were to be excluded from the congress. Even the scientific community could not then forget the war-time hatred and bitterness. Wiener did not care about the politics of that time. He was only too eager to resume scientific contact with Europe after such a long lull. He still had an emotional attachment for Europe and he wanted to be independent. Before the start of the congress in September, he spent some time working with the French analyst, Maurice Fréchet. It was during this period that he discovered, independently of the Polish mathematician Stefan Banach [33], axioms for vector spaces. However, Wiener did not continue to do much research in this field of Banach spaces.

In 1926, Wiener married Margaret Engemann who, at the age of fourteen, migrated to America from Germany with her widowed mother and sisters. She was to give him the emotional support and understanding that was so essential to him in his climb to conquer challenging mathematical heights. He was a keen traveller. Before marriage, he had been to England and Europe several times, at times by himself and at other times with his sisters. Shortly after his marriage, he went to Göttingen as a Guggenheim Fellow. Because of her teaching job, the newly-wedded wife could only join him in Europe later. With his wife and two daughters, Wiener spent 1931-1932 at Cambridge University and 1935-1936 at Tsing Hua University in Peking. His stay at Tsing Hua was partly arranged by his former Ph.D. student, Y.W.Lee [34], an electrical engineer by training with whom he had patented an invention.

In 1933, for his work on generalized harmonic analysis and Tauberian theorems, Wiener shared with Marston Morse [35] the American Mathematical Society's Bôcher prize, which is awarded every five years to American mathematicians for outstanding research.

He had strong views against the construction and use of nuclear bombs, and resented the intrusion of the military establishment into the laboratory in an attempt to control the dissemination of knowledge. His work on prediction and computing machines gave him a vision of a future society in which automation would replace a considerable amount of human labour. The moral issues arising out of automation loomed ominously over his mental horizons, and he felt it his duty to speak about the social problems of automation to union officials and administrators.

In 1946, Wiener went to France to attend a conference on harmonic analysis. When he reached Paris, Freymannn of the French publishing company, Hermann et Cie, met him and solicited for a writing up of his ideas on control and communication. Here was a good opportunity for him to present a unified view-point on the scattered but related ideas that have been nutured over a period of more than ten years. Two years later, his well-known book *Cybernetics* [46] was published.

The word "cybernetics" had been coined by Wiener himself from the Greek word for steersman and he was then unaware that the French physicist André Ampère [47] had used the same word before, but in a sociological sense. The seeds of this new discipline were sown as far back as 1933 when he came to know the Mexican physiologist Arturo Rosenblueth [48] who was at Harvard Medical School before his return to Mexico in 1944. During those pre-war years, Rosenblueth conducted a monthly series of discussion meetings in which a scientific paper would be read and a free and unrestrained discussion, with emphasis on methodology, would ensue. Wiener was introduced to those meetings by the Mexican physicist, Manuel Sandova Vallarta [49] of M.I.T., and subsequently became an active participant. His fruitful collaboration with Rosenblueth continued through the war. After the war, they took turns to visit each other.

There was an exchange of ideas on computing machines between Wiener and John von Neumann [50] who was then heading the Computer Project at the Institute for Advanced Study. In the late winter of 1943 - 1944, they convened at Princeton what might be called the inaugural meeting of cybernetics. It prepared a common ground for a multitude of disciplines. Subsequently, in 1946, the Macey Foundation series of conferences on cybernetics was launched in a meeting in New York and lasted for several years. Cybernetics was thus firmly planted. It has grown and developed since then.

In 1944, Wiener was in Mexico to attend a meeting of the Mexican Mathematical Society. This was the first of many visits to Mexico in the course of his joint research with Rosenblueth. In 1953, Wiener was invited by the Indian government to undertake a seven-week lecture trip in India. He published in 1959 a novel *The tempter* in which the main character was inspired by Heaviside.

M.I.T. appointed Wiener as Institute Professor in 1959. This was a distinguished position which allowed him access to any department in the institute. He retired in 1960. He was to receive from President Johnson the National Medal of Science shortly before his death on 18 March 1964.

Towards synthesis. Running through Wiener's multifarious contributions is a common thread of motivation and intuition that unwinds from his interest in the applications of mathematics to physics and engineering. In electrical engineering, Heaviside was his idol. At heart, Wiener was a pure mathematician (perhaps his initial training as a logician made exacting demands on his thinking). As Levinson puts it, " ... once Wiener tackled a problem, his treatmentwas rigorous, general and aesthetic. Perhaps the framing of his theories in the full generality and abstraction of the Lebesgue integral delayed their accessibility to the engineer. Yet he could do it no other way". [51].

Out of a problem of integration in function spaces arose his greatest work on Brownian motion and random processes. When he contemplated this problem as an instructor at M.I.T. the physical theory of Brownian motion was thought to be finished. It was his physical insight that made him revive this subject and build a whole edifice whose ramifications continue to spread and multiply more than fifty years later. Mark Kac [52] points out that even Wiener's brief but significant work in potential theory done around the same time as that on Brownian motion has turned out to be closely related to the latter work. It is worthwhile noting that his probabilistic analysis was done more than ten years before the axiomatic formulation of probability by the Russian mathematician A.N.Kolmogorov [53] in 1933.

The breakdown of classical harmonic analysis in the study of white light led Wiener to develop a "generalized" harmonic analysis. Some of his ideas have since been incorporated into works on optics. While he was in Göttingen as a Guggenheim Fellow in 1926-1927, the British mathematician A.E. Ingham [54] pointed out to him the similarity between the so-called Tauberian theorems and certain results in Wiener's new analysis. Tauberian theorems were hitherto rather scattered results to which Hardy and Littlewood made numerous contributions, but there did not seem to be a general theory. Wiener then saw that his generalized analysis provided the tool for a general theory of Tauberian theorems. At the same time, his methods could also be used to give a simpler proof of the prime-number theorem [55].

More than ten years later, his generalized harmonic analysis again found its way into his war-time work on prediction theory. In his 1942 classic *The extrapolation, interpolation and smoothing of stationary time series*, he introduced statistical concepts into the study of the transmission of messages and noise, thereby laying the foundation for communication and control theory. At that time, he was unaware that Kolmogorov had earlier published similar mathematical ideas on prediction theory. But only Wiener had made significant use of the theory in engineering. His study of linear and non-linear filters in electrical engineering stirred up deeper questions on the learning, self-organizing and reproducing abilities of natural systems.

A product of war-time work was the information theory developed by Wiener and Claude E. Shannon [56] . The former approached it via the electric circuit, carrying a continuous current, while the latter considered a message as a discrete sequence of yeses and noes. The feedback mechanism that was discovered in conjunction with his project on fire-control reminded Wiener of a closely related phenomenon in biology - that of homeostasis, the process by which the internal environment of a living organism is maintained at a healthy level. This conglomeration of ideas found synthesis in his book on cybernetics. The underlying philosophy of this synthesis is found in the following passage from his autobiography. [57].

"We are swimming upstream against a great torrent of disorganization, which tends to reduce everything to the heat-death of equilibrium and sameness described in the second law of thermodynamics. What Maxwell [58], Boltzmann [59] and Gibbs [60] meant by this heat death in physics has a counterpart in the ethics of Kirkegaard [61] who pointed out that we live in a chaotic moral universe. In this, our main obligation is to establish arbitrary enclaves of order and system. These enclaves will not remain there indefinitely by any momentum of their own after we have once established them. Like the Red Queen, we cannot stay where we are without running as fast as we can.

"We are not fighting for a definitive victory ... indefinite future. It is the greatest possible victory to be, to continue to be, and to have been. No defeat can deprive us of the success of having existed for some moment of time in a universe that seems indifferent to us.

"This is no defeatism, it is rather a sense of tragedy in a world in which necessity is represented by an inevitable disappearance of differentiation. The declaration of our own nature and the attempt to build up an enclave of organization in the face of nature's overwhelming tendency to disorder is an insolence against the gods and the iron necessity that they impose. Here lies tragedy, but here lies glory too".

A survey and evaluation of his many contributions to our knowledge is given in a special memorial issue of the Bulletin of the American Mathematical Society [62] and in a collection of selected papers [63]. The latter contains his most important mathematical works and gives a cross-section of his achievements.

Among the biological research which he carried out (in collaboration with others and, in particular, with Rosenblueth) or stimulated is that on biological regulation, characterization of the electroencephalogram as a time series and prosthetic devices. However, after the publication of his book on cybernetics he did not keep abreast of the advances in molecular biology while others took up the challenges posed by the flow of intra-and inter-cellular information.

Wiener's success is that he has not only planted the seeds of knowledge but also fertilized the ground in which seeds sown by others may germinate and grow. If, eventually, his fundamental achievements become particular undergrowths in a luxuriant forest, let us bear in mind these words of Kac," The fate of all great work is to be subsumed; the more attention it attracts the greater the chances of becoming engulfed in a cascade of generalizations and extensions." [64].

Finally, Wiener has this to say about the creative urge in mathematics.

" Mathematics is too arduous and unviviting a field to appeal to those to whom it does not give great rewards. These rewards are of exactly the same character as those of the artist. To see a difficult, uncompromising material take living shape and meaning is to be Pygmalion, whether the material is stone or hand, stonelike logic. To see meaning and understanding come where there has been no meaning and no understanding is to share the work of a demiurge. No amount of technical correctness and no amount of labour can replace this creative moment , whether in the life of a mathematician or in that of a painter or musician. Bound up with it is a judgement of values, quite parallel to the judgement of values that belongs to the painter or musician. Neither the artist nor the mathematician may be able to tell you what constitutes the difference between a significant piece of work and an inflated trifle; but if he is never able to recognize this in his own heart, he is no artist and no mathematician." 65

Noted and references


[1] Godfrey Harold Hardy (1877-1947), British mathematician; studied at Cambridge; worked at Cambridge and Oxford; contributed to analysis and analytic theory of numbers.

[2] Norbert Wiener, *Ex-prodigy*, Simon and Schuster, 1953, M.I.T. Press, 1964.

[3] Norbert Wiener, *I am a mathematician*, Victor Gollancz, London, 1956.

[4] Leo Tolstoy (1828-1910), Russian novelist, moral philosopher and social reformer; studied at Kazan University, fought in Crimean War; well-known for his novels (especially *War and Peace*, *Anna Karenina*) and for his philosophy of humanism.

[5] Moses Maimonides (1135-1204), Jewish philosopher; lived in Cordoba (Spain); migrated to Cairo (Egypt) after Mohammedan invasion; became royal physician; did definitive work on the Judaic laws of the Talmud.

[6] Edmund Vermilye Huntington (born 1874), American logician; studied at Harvard and Strassburg; worked at Harvard; conributed to postulational theory of mathematics.

[7] Josiah Royce (1855-1916), American philosopher; studied at California, Leipzig, Göttingen, Johns Hopkins; worked at California, Harvard; contributed to philosophy and psychology.

[8] Maxime Bôcher (1867-1918), American mathematician; studied at Harvard, Gottingen; worked at Harvard, Paris; contributed to theory of differential equations.

[9] Oswald Veblen (born 1880), American mathematician; studied at Iowa, Harvard, Chicago; worked at Iowa, Chicago, Princeton, Institute of Advanced Study (Princeton); contributed to geometry and topology.

[10] John Wesley Young (1879-1932), American mathematician; studied in Germany and at Ohio State University, Cornell University; worked at Northwestern, Princeton, Illinois Kansas, Chicago, Dartmouth College; contributed to postulational theory of geometry.

[11] Bertrand Russell (1872-1970), British logician and philosopher; Nobel laureate in literature (1950);studied and worked at Cambridge; contributed to logic and foundation of mathematics, philosophy, pedagogy and literature.

[12] Henry Frederick Baker (1866-1956), British mathematician; studied and worked at Cambridge; contributed to geometry, theory of algebraic and multiply periodic functions, theory of differential equations, dynamical astronomy.

[13] John Edensor Littlewood (born 1885), British mathematician; studied at Cambridge; worked at Manchester, Cambridge; contributed to theory of functions.

[14] James Mercer (1883-1932), British mathematician; studied and worked at Liverpool and Cambridge; contributed to theory of integral equations.

[15] Albert Einstein (1879-1955), German-born American physicist; Nobel laureate (1922); worked in Berne, Zürich, Prague, Berlin, Princeton; contributed to theories of Brownian motion, photo-electricity, special relativity, general relativity.

[16] Niels Bohr (1885-1962), Danish physicist; Nobel laureate (1922); studied at University of Copenhagen; worked at Cambridge; Manchester, Copenhagen; was in Washington and Los Alamos during Second World War; contributed to quantum theory and nuclear physics.

[17] Kurt Gödel (born 1906), Czech-born American logician; studied at Vienna; worked at Vienna and Institute of Advanced Study (Princeton); contributed to mathematical logic and foundations of mathematics.

[18] Edmund Landau (1877-1938), German mathematician; studied in Berlin and Munich; worked at Berlin, Gottingen; contributed to analytic number theory.

[19] David Hilbert (1861-1943), German mathematician and theoretical physicist; contributed to number theory, set theory, logic, geometry, integral equations and mathematical physics.

[20] Edmund Husserl (1859-1938), German philosopher; studied at Leipzig, Berlin, Wien, Vienna; worked at Halle, Göttingen, Freiburg im Breisgan; contributed to philosophy and phenomenology.

[21] For some years before the Second World War, Harvard University implemented the system of Docent Lectures along that of the German system. Every Harvard Ph.D. was granted the right to give a series of lectures in his own field without pay. Harvard students would be free to attend them but they would not be taken into account for a degree.

[22] Vito Volterra (1860-1940), Italian nathematician; child prodigy; studied at Florence and Pisa; worked at Pisa, Turin, Rome, Paris; fought in First World War with the Air Force; lived mainly outside Italy after 1932; contributed to analysis, theory of integral equations , mathematical biology, mathematical physics.

[23] William Fogg Osgood (1864-1943), American mathematician; studied at Harvard, Göttingen, Erlangen (Germany); worked at Harvard and Peking; contributed to theory of functions.

[24] Henri Lebesgue (1875-1941), French mathematician; studied at the Ecole Normale Supérieure; worked at Rennes, Poitiers, Paris; contributed to analysis and theory of Fourier series.

[25] Maurice René Fréchet (born 1878), French mathematician; studied in Lycée Saint Louis; worked at Nantes, Rennes, Poitiers, Strasbourg; contributed to set theory, analysis, topology, probability, mathematical statistics.

[26] Isaac Albert Barnett (born 1894), British-born American mathematician, studied at Chicago; worked at Washington, Harvard, Saskatchewan, Cincinati; contributed to calculus of variations, integral equations, classical theory of numbers.

[27] Norman Levinson, "Wiener's life", *Bull. American Mathematical Society*, Vol. 72, No. 1, Part II, 1966.

[28] Oliver D. Kellogg (1878-1932), American mathematician; worked at Harvard; contributed to theory of integral and differential equations, potential theory.

[29] Norman Levinson (born 1912), American mathematician; studied and worked at M.I.T., contributed to theory of complex variables, transform theory, differential equations.

[30] Oliver Heaviside (1850-1925), British mathematical physicist; self-taught; received Ph.D (hon.) from Göttingen University; worked as telegrapher; contributed to electromagnetic theory, telegraphy.

[31] In 1897, A.Tauber proved that if $a_0 + a_1x + a_2x^2 + \ldots$ tends to A as $x \to 1-0$, and $a_n = o(1/n)$ (i.e. $na_n \to 0$), then $a_0 + a_1 + a_2 + \ldots = A$. Hardy and Littlewood replaced the hypothesis $a_n = O(1/n)$ by the hypothesis $a_n = O(1/n)$ (i.e. there is a constant K such that $n|a_n| \leqslant K$ for $n \geqslant n_0$).

[32] The International Mathematical Congress is held every four years by the International Mathematical Union. At each congress, one or more mathematicians (below the age of 40) would be awarded the prestigious Fields Medal for significant contributions to mathematics. The Congress of 1974 has held in Vancouver (Canada), and the recipients of the Fields Medal at that congress were the American Mumford and the Italian Bombieri.

[33] Stefan Banach (1892-1945), Polish mathematicain; studied and worked at Lwow (Poland); contributed to functional analysis, topology.

[34] Yuk-Wing Lee (born 1904); Chinese-born American electrical engineer; studied at M.I.T.; worked in New York, Shanghai, Tsing Hua; now at M.I.T.: contributed to communication engineering.

[35] Marston Morse (born 1892), American mathematician; studied at Colby College and Harvard; worked at Harvard, Cornell, Brown, Institute of Advanced Study (Princeton); contributed to theory of dynamical systems, analysis in the large, function theory.

[36] Raymond E.A.C.Paley (1907-1933), British mathematician; worked at Cambridge and M.I.T.: was killed in a skiing accident in the Canadian Rockies; contributed to theory of Fourier transforms.

[37] See [27].

[38] Otto Szász (born 1884), Hungarian-born American mathematician; studied at Budapest; worked at Frankfurt, M.I.T., Cincinnati; contributed to analysis, theory of approximation, number theory.

[39] Hans Rademacher (born 1892), German-born American mathematician; studied at Göttingen; worked at Berlin, Hamburg, Breslau, Pennsylvania; contributed to analysis and analytic number theory.

[40] George Pólya (born 1887), Hungarian-born American mathematician; worked in Göttingen, Zürich, Stanford; contributed to analysis, probability theory, number theory, applied mathematics.

[41] Gabor Szegö (born 1895), Hungarian-born American mathematician; studied at Vienna; worked at Berlin, Königsberg, Washington, Stanford; contributed to pure and applied mathematics.

[42] Karl Menger (born 1902), Austrian-born American mathematician; studied at Vienna; worked at Amsterdam, Vienna, University of Notre Dame and Illinois Institute of Technology; contributed to geometry, algebra, analysis.

[43] Eberhard Hopf (born 1902), Austrian mathematician; studied in Germany and at Harvard; worked at M.I.T. and in Germany.

[44] Julian Himely Bigelow, American electrical engineer and computer scientist; worked at International Business Machines; first chief engineer of Institute for Advanced Study Electronic Computer Project.

[45] Harold Urey (born 1893), American chemist; Nobel laureate (1934); studied at Montana and California;worked at Montana, Copenhagen, Columbia, Chicago, California; contributed to nuclear chemistry, chemistry of the origins of earth, meteorites, moon and solar system.

[46] Norbert Wiener, *Cybernetics*, M.I.T. Press, Cambridge, Massachusetts, 1961.

[47] André Marie Ampère (1775-1836), French mathematical physicist; worked in Lyons and Paris; contributed to electomagnetic theory.

[48] Arturo Rosenblueth (born 1900), Mexican physiologist; studied at Mexico, Paris; worked at Harvard, Mexico; contributed to neurophysiology and electrophysiology.

[49] Manuel Sandova Vallarta (born 1899), Mexican theoretical physicist; studied at M.I.T.; worked at M.I.T. and Mexico; contributed to theory of cosmic radiation and relativity.

[50] John von Neumann (1903-1957), Hungarian-born mathematician and theoretical physicist; studied in Budapest, Berlin, Zürich, Göttingen; worked at Berlin, Hamburg, Institute for Advanced Study (Princeton); contributed to set theory, algebra,analysis, game theory, computer science, mathematical physics. See Y.K.Leong, "Notes on mathematicians: 3. John von Neumann", *Mathematical Medley* 3(1975), 90-106.

[51] Norman Levinson, "Introduction and some comments on Wiener's selected papers", in *Selected papers of Norbert Wiener*, M.I.T. Press, Cambridge, Massachusetts, 1964, pp. 1,2.

[52] Mark Kac (born 1914); Polish-born American mathematician; studied in Lwow (Poland); worked in Lwow, Cornell University Rockefeller University; contributed to theory of probability, mathematical statistics, analysis, number theory, statistical mechanics.

[53] Andrei Nikolaevich Kolmogorov(born 1903), Russian mathematician; studied at Moscow; worked at Paris, Moscow; contributed to analysis, probability,mathematical statistics.

[54] Albert Edmund Ingham (1900-1967), British mathematician; studied at Cambridge;worked at Göttingen, Leeds, Cambridge; contributed to analysis and analytic number theory.

[55] The Prime Number Theorem states that if $\pi(x)$ is the number of primes not exceeding the real number x, then $(\pi(x)\log x)/x \to 1$ as $x \to \infty$. This was conjectured by Legendre (1752-1833) and Gauss (1777-1855), and was proved by Hadamard (1865-1963) and, simultaneously, de la Vallée Poussin (1866-1962) using the theory of entire functions. An elementary number-theoretic proof was given by Selberg and Erdös in 1947.

[56] Claude Elwood Shannon (born 1916), American applied mathematician; studied at Michigan and M.I.T.; worked in Bell Telephone Laboratories; now at MIT; contributed to communication theory, mathematical cryptography, theory of computing machines.

[57] *I am a mathematician,* pp. 324,325. See 3.

[58] James Clark Maxwell (1831-1879), British mathematical physicist; studied at Edinburgh, Cambridge; worked at Aberdeen, London and Cambridge; contributed to electromagnetic theory, statistical mechanics; thermodynamics.

[59] Ludwig Eduard Boltzmann (1844-1906), Austrian theoretical physicist; studied at Vienna, worked at Vienna, Graz (Austria), Munich, Leipzig; contributed to electromagnetic theory, thermodynamics, statistical mechanics.

[60] Josiah Willard Gibbs (1839-1903), American mathematician physicist; studied at Yale, Paris, Berlin, Heidelberg; worked at Yale; contributed to thermodynamics, statistical mechanics, optics.

[61] Søren Kirkegaard (1813-1855), Danish theologian and philospher; studied at Copenhagen and entered the pastoral seminary; rebelled against the philosophical tradition of his time; made little impact during his life-time; originator and fore-runner of modern existentialist philosophy.

[62] *Bull. American Mathematical Society*, Vol. 72, No. 1, Part II, 1966.

[63] *Selected papers of Norbert Wiener*, with contributions by Y.W.Lee, N.Levinson, W.T.Martin, M.I.T. Press, Cambridge, Massachusetts, 1964.

[64] See [62] , p. 68.

[65] See [2] , p. 212.

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*'We do not claim for mathematics the prerogative of a Queen of Science, there are other fields which are of the same or even higher importance in education. But mathematics sets the standard of objective truth for all intellectual endeavours; science and technology bear witness to its practical usefulness. Besides language and music it is one of the primary manifestations of the free creative power of the human mind, and it is the universal organ for world-understanding through theoretical construction. Mathematics must therefore remain an essential element of the knowledge and abilities which we have to teach, of the culture we have to transmit, to the next generation. Only he who knows what mathematics is, and what its function in our present civilization is, can give sound advice for the improvement of our mathematical teaching.'*

*Hermann Weyl (1885 - 1955)*