MANPOWER PLANNING*

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1 What is Manpower Planning?

Manpower planning aims to match manpower supply and demand in an organisation, big or small. The demand may result from the needs of the community, the expansion of a company, or from vacancies due to wastage. Manpower may be supplied from recruitment, job retraining, or promotion between grades.

It is conwenient to distibguish between PUSH and PULL Manpower Planning. As an example of the PUSH concept in educational planning , we may cite the 1963 Robbins report which was responsible for a tremendous growth of higher education in the U.K.:

> " Courses of higher education should be available for all students qualified to pursue them and who wish to do so".

By contrast, the PULL concept is explicit in the statement of Lord Crowther-Hunt, Minister of State and responsible for tertiary education, which appeared in the Times Higher Education Supplement, May 16, 1975:

> " Manpower Planning is to be increased. We should do our best to produce the sort of educated people the nation needs".

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* This is the text of a talk organised jointly by the Singapore Mathematical Society and the Department of Mathematics, University of Singapore, on 31st March 1976.

2 Manpower Flow Models

Mathematical models for Manpower Planning have been extensively developed in the last few years [1]. The movement of manpower is usually described as a manpower flow and we distinguish between a

push flow in which the impetus comes from the present status and location of the individual

and a

pull flow for which the reason for the move lies at the destination.

For example, a resignation of an employee is a push flow while a recruitment to fill a vacancy is a pull flow.

There are two main classes of manpower flow models. The first is the *Markov model*, which is essentially a push model. In this, recruiting and promotion policies are given and the wastage known; the model is used to predict the manpower numbers. By contrast, the *renewal model*, essentially a pull model, determines the necessary recruiting and promotions needed to give the required manpower numbers, with wastage again assumed known. The models may either be deterministic or stochastic, although only the former will be considered in this article,

3 Markov Model of University Staff

By way of illustration consider a manpower flow model for the staff of a University faculty. Let us suppose that heads are counted on January 1st and the discrete variable t is used for time :

date	1/1/76	1/1/77	1/1/78	...
t	0	1	2	...

We also allow for 5 grades; tutor , lecturer, senior lecturer, reader, professor, denoted by j as follows :

grade	T	L	SL	R	P
j	1	2	3	4	5

We introduce the following notation:

(1) $n_j(t)$ = number of staff at grade j at time t

(2) $r_j(t)$ = number of staff recruited at grade j at time t

(3) $p_{ij}(t)$ = proportion of staff at grade i at time t-1 who are at grade j at time t

Note that (3) implies that

(4) $1 - \sum_j p_{ij}(t) = w_i(t)$ = proportion of staff at grade i at time t-1 who are wasted.

The fundamental first order difference equation for the model is

(5) $n_j(t) = \sum_i n_i(t-1)\, p_{ij}(t) + r_j(t), \quad t \geqslant 1,$

with $n_j(o)$ given. For given recruiting and promotion, the staff numbers can be determined at time t from the numbers at time t-1. The model is thus a Markov or push model.

The solution of the difference equation is trivial, In the simple case when r_j and p_{ij} are independent of t, we can rewrite (5) as

(6) $$\underset{\sim}{n}(t) = \underset{\sim}{n}(t-1)\,\underset{\sim}{P} + \underset{\sim}{r}, \quad t \geqslant 1$$

where $\underset{\sim}{n}(t)$ and $\underset{\sim}{r}$ are row vectors and $\underset{\sim}{P}$ a matrix with elements p_{ij}. The solution of (6) is

(7) $$\underset{\sim}{n}(t) = \underset{\sim}{n}(o)\,\underset{\sim}{P}^{\,t} + \underset{\sim}{r} \sum_{s=0}^{t-1} \underset{\sim}{P}^{\,s}$$

Provided the row sums of $\underset{\sim}{P}$ are less than unity, the solution converges to the steady state

(8) $$\underset{\sim}{n}(\infty) = \underset{\sim}{r}\,(\underset{\sim}{I}-\underset{\sim}{P})^{-1}$$

This elementary analysis is of course familiar as an example of a birth and death process [1] .

For a numerical example (adapted from [1]), let

$$
(9)\qquad \underset{\sim}{P} = \begin{array}{c} \\ T \\ L \\ SL \\ R \\ P \end{array}
\begin{array}{c}
\begin{array}{ccccc} T & L & SL & R & P \end{array} \\
\left[\begin{array}{ccccc}
.65 & .20 & 0 & 0 & 0 \\
0 & .70 & .15 & 0 & 0 \\
0 & 0 & .70 & .15 & 0 \\
0 & 0 & 0 & .85 & .10 \\
0 & 0 & 0 & 0 & .95
\end{array}\right]
\end{array}
\begin{array}{c} \text{waste} \\ .15 \\ .15 \\ .10 \\ .05 \\ .05 \end{array}
$$

Thus each year, 70% of the lecturers remain lecturers, 15% are promoted to senior lecturers, the remaining 15% being wasted (resignations, death, etc).

If we choose a constant recruiting policy of 9 tutors and 3 lecturers each year, we can easily use (5) or (7) to determine the staff numbers in the various grades in each year starting with any given initial values. The results of such a computation are given in Table 1 for initial staff values given by the row $\underset{\sim}{n}(o)$.

Table 1

Numerical Values for a Markov Model of University Staff

	T	L	SL	R	P	Total
$\underset{\sim}{r}$	9	3	0	0	0	12
$\underset{\sim}{n}(o)$	47	35	18	12	6	118
$\underset{\sim}{n}(1)$	40	37	18	13	7	115
$\underset{\sim}{n}(2)$	35	37	19	14	8	113
$\underset{\sim}{n}(5)$	28	33	21	16	11	109
$\underset{\sim}{n}(10)$	26	29	19	18	16	108
$n(\ )$	26	27	16	16	33	118

There are several interesting points illustrated by this example.

(1) The recruiting numbers and the initial staff numbers were purposely chosen so that the total staff in the steady state (viz. 118) would be the same as that initially.

(2) The steadystate is not a good guide to the transient situation in the first few years. The early trend is for the total staff to decrease and it would be hard to guess that ultimately the professors would become so dominant! As the convergence is slow, the transient numbers are of much more practical importance than the never-to-be-reached steady state.

(3) Because demotions are not suppose to occur, $\underset{\sim}{P}$ is an upper triangular natrix whose eigenvalues are simply the diagonal elements. The largest eigenvalue 0.95, being close to 1, accounts for the slow convergence.

(4) In practice a much more complicated Markov model would be needed to be of use in predicting University staff numbers. A typical and important constraint might be that the number of senior lecturers and above should be less than 40% of the total staff.

4 Age Distributions

In many organisations, such as a Public Service, recruiting promotion and wastage are strongly age dependent and a pull model is more appropriate in determining how to meet the manpower numbers required at different levels. One important aspect is the desirability of maintaining a *stationary age distribution* for which the career opportunities for recruits are the same year by year. Unfortunately the situation is usually unstable in the way illustrated in Fig. 1. In year 1970, the career opportunities for those recruited as 20 year olds are poor while in year 2000 they would be good.

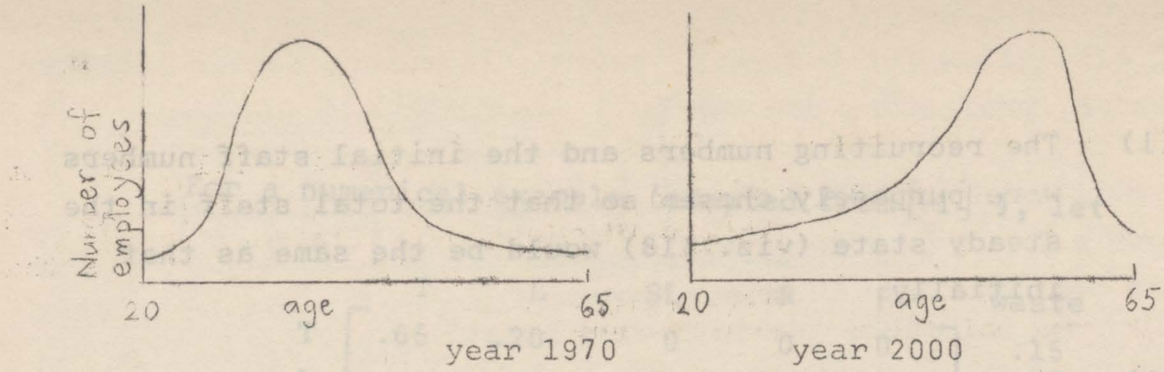

Figure 1. Change of Age Distribution with Time

By way of a simple illustration, consider an organisation with one grade of employee and a minimum age of 20 and a compulsory retiring age of 65.

We suppose that heads are counted on January 1st, that recruiting takes place on this same date, while retirements, resignations deaths, etc all occur on December 31st. Let us introduce the notation:

(10) $n(x,t)$ = number of staff age x time t,

(11) $r(x,t)$ = number of recruits age x at time t,

(12) $w(x,t)$ = number of staff age x retiring at time t.

Then the fundamental first-order difference equation describing the process is

(13) $$n(x,t) = n(x-1,t-1) - w(x-1,t-1) + r(x,t), \quad 21 \leq x \leq 65,$$

(14) $$n(20,t) = r(20,t)$$

(15) $$w(65,t) = n(65,t)$$

For simplicity we consider the case of an organisation with constant growth factor k (for $k > 1$ the organisation is expanding and $k < 1$ contracting). For a stationary age distribution we then require

(16) $$n(x,t) = n(x)k^t$$

(17) $$r(x,t) = r(x)k^t$$

(18) $$w(x,t) = (x)n(x)k^t$$

where (x) is the proportion of those age x who are wasted.

Equation (13) simplifies to

(19) $$kn(x) = n(x-1) - \alpha(x-1)n(x-1) + kr(x)$$

with the solution

(20) $$n(x) = \sum_{y=20}^{x} r(y) \prod_{z=y}^{x-1} k^{-1}\left[1 - \alpha(z)\right]$$

As a pull model, this solution could be used to determine the recruiting needed to give a desired steady age distribution for assumed wastage proportions. The model can be extended to take into account different grades and has been used [2] to suggest possible ways of achieving a desired stationary age distribution for a given initial distribution. Possibilities for this include recruitment of senior personnel on the one hand or early retirement schemes on the other.

5 Vacancy Flow Model

It is interesting to note that there is a duality between manpower flows and vacancy flows. Thus a resignation can be considered as the recruit of a vacancy, a recruitment as the wastage of a vacancy, and a promotion as the demotion of a vacancy. Thus in a sense a renewal manpower flow model can be considered as a Markov vacancy flow model. This duality is conceptually attractive for an organisation such as a Public Service for which a pull model is appropriate and for which the vacancy flow rates are fairly predictable. Such a model is presently being considered by a research student at the University of Adelaide. By itself the model cannot expect to have the answer to the many and uncertain problems of manpower planning but it is hoped that it will be of some assistance to management in the important manpower decisions it has to make.

References

1 Bartholomew, D. J., *Stochastic Models for Social Processes*, Wiley (2nd ed, 1973).

2 Kennay, G.A., Morgan, R. W. and Ray, K. H., " The use of steady state models for career planning in an expanding organisation" in *Manpower planning models*, edited by Clough, D.J., English University Press (1974).

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' *Everything of importance has been said before by somebody who did not discover it.* '

Alfred North Whitehead

(1861 - 1947)