# NOTES ON MATHEMATICIANS

## 3. John von Neumann (1903-1957)

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John von Neumann belonged to the pantheon of universalists in mathematical thought. He was a rare example of a breed of great mathematicians who made deep and far-reaching contributions to diverse areas of mathematics and mathematical physics. This breed includes Isaac Newton [1] in the seventeenth century, Leonhard Euler [2] and Joseph-Louis Lagrange [3] in the eighteenth century, Carl Friedrich Gauss [4] and Henri Poincaré [5] in the nineteenth century, and David Hilbert [6] , Hermann Weyl [7] and Norbert Wiener [8] in the present century. The first three decades of this century witnessed a remarkable phenomenon in the history of mathematics and physics: the achievements of epoch-making breakthroughs that heralded the arrival of modern mathematics and modern physics, and the emergence of Göttingen [9] as the centre of mathematical research. This phenomenon owes a debt to the fortuitous simultaneous appearance of a myriad of brilliant young stars in the galaxy of mathematicians. And one of the most brilliant of these young stars was John von Neumann.

European prelude. He was born on 28 December 1903 in Budapest, Hungary, as the eldest of three sons in a well-to-do Jewish family. His father, a banker, received from Emperor Franz Josef in 1913 a minor hereditary title of nobility — Margittai (which means "of Margitta" and which was Germanized by young von Neumann to "von"). As the Hungarians put the family name first, von Neumann's full Hungarian name was Margittai Neumann Janos (meaning John Neumann of Margitta).

As a boy, his intellectual abilities revealed a phenomenal speed in learning and thinking. When he was 6 years old, he could divide two eight-digit numbers mentally. He had mastered the calculus by 8, and had read Emile Borel's [10] *Theorie des Fonctions* by 12. On the linguistic side, he was proficient enough at the age of 6 to often joke with his father in classical Greek.

Like Poincaré and(to a lesser degree Euler (see [46] ), von Neumann possessed a fantastic power of absolute recall. He could quote back verbatim a book or aticle read once, even years later. His powerful memory together with his early love of history gave him a deep and encyclopedic knowledge of, among other subjects which he studied for diversion, Byzantine history, and the details of the trial of Joan of Arc and and of the battles of the American Civil War.

In 1919, Bela Kun [11] staged a communist uprising and ruled Hungary for 130 days. A month after he seized power, the von Neumanns left Hungary for Venice. They returned home almost two months after Bela Kun was driven out. This event affected young von Neumann deeply and instilled in him a fear and dislike of Russia.

From 1911 to 1921, he attended the Lutheran gymnasium [12] in Budapest which was one of the best schools in Hungary. His school report cards showed that he obtained A grades in all subjects except in geometrical drawing (for which he got B), writing (B), music (B), physical education (C) and behaviour (sometimes A, but more often B). While in school, he came under the guidance of a dedicated teacher, Laszlo Rátz. The first research paper published by von Neumann when he was barely 19 was a joint paper written with the well-known young mathematician, Michael Fekete [13] .

When von Neumann was 17, his father asked Theodore von Kármán [14] to dissuade him from taking up mathematics as a vocation, for the obvious reason that mathematics did not offer a lucrative career. The compromise solution suggested by von Kármán was chemistry. So in 1921, von Neumann enrolled in the University of Budapest. But from 1921 to 1923, he was in Berlin studying chemistry under Fritz Haber [15] and mathematics under Erhard Schmidt [16]. In 1923, he went to the Swiss Federal Institute of Technology (Eidgenössische Technische Hochschule) in Zürich, from which he obtained a degree in chemical engineering in 1925. While in Zürich, he was in contact with the mathematicians Hermann Weyl and George Pólya [17], both of whom were in their late thirties. From 1923 to 1925, von Neumann published three research papers on set theory. On 12 March 1926, he received from the University of Budapest his doctorate *summa cum laude* [18] in mathematics with minors in experimental physics and chemistry.

From 1926 to 1929, he was a Privatdozent [19] in mathematics at the University of Berlin, and was in Hamburg in 1929. By the age of 24, his work on algebra, set theory and quantum mechanics won him international acclaim as a great mathematician, and he was referred to as a "youthful genius" [47]. He often said that one's mathematical powers decline after the age of 26 though this is compensated by "a certain prosaic shrewdness developing from experience" [48].

Young von Neumann was interested in Hilbert's ideas on proof theory and mathematical physics, and so became a frequent visitor to Göttingen in 1924. He had long discussions with Hilbert in Hilbert's garden or study. Yet the contrast between the two mathematicians was strikingly sharp: Hilbert was in his sixties, more than forty years older than von Neumann; the former was "slow to understand" while the latter had, according to Lothar Nordheim [20], "the fastest mind I ever met" [48]. It seems that while in Göttingen, von Neumann already exhibited a passion for mechanical toys.

New World. In January, 1930, von Neumann was invited to Princeton University together with the physicist and fellow Hungarian, Eugene Wigner [21] . It was at the suggestion of the physicist, Paul Ehrenfest [22] , that this invitation was sent by Oswald Veblen [23] , then a professor at Princeton. In 1931, von Neumann became a permanent professor, and in 1933, he joined the mathematics faculty of the Institute for Advanced Study, which was then located at Princeton's Fine Hall. He held the professorship at the Institute for the rest of his life. The Institute was incorporated on 20 May 1930. When its mathematics faculty was set up, the first six professors were James Alexander [24] , Albert Einstein [25] , Marston Morse [26], Veblen, von Neumann and Weyl.

During the early thirties, the oppressive political atmosphere in Europe (with overtones of racial persecution) began to be felt by scientists and intellectuals many of whom, within the next few years, emigrated in droves and found refuge west of the Atlantic. This exodus of talent from the Old World shifted the centre of science and technology to the New World. From this transfer of scientific and intellectual leadership Europe has not recovered.

While the clouds of political and social uncertainty descended on Europe, von Neumann prepared to settle to a new way of life in America, leaving behind him half his life time of European experience. Although he missed the informal and congenial atmosphere of the Viennese café, he adapted to and enjoyed the new social life. Unlike many European emigrés, he did not feel like a refugee. As Paul Halmos [27] writes [49] : "He was a cosmopolite in attitude and a U.S. citizen by choice." He was known to most people as Johnny and to some as Jancsi. He frequently threw parties at his home, often to introduce temporary members of the Institute to their colleagues. His powerful memory enabled him to store up a large collection of anecdotes, limericks and amusing incidents which was a source of entertainment to his guests.

In 1930, von Neumann and Marietta Kövesi were married, and their daughter Marina was born in 1935. Marina had a distinguished scholastic record, became a professor of economics at the University of Pittsburg and was appointed by President Nixon to the Council of Economic Advisers. In 1937, the von Neumanns were divorced, and he married Klára Dán in 1938. He taught Klára mathematics and she became a programmer for the Los Alamos Scientific Laboratory which became a war-time centre of scientific activity.

In 1937, von Neumann was the American Mathematical Society's Gibbs Lecturer and Colloquium Lecturer, and was awarded the Bôcher Prize for his work on almost periodic functions on groups. By the mid thirties, he was deeply interested in the study of supersonic and turbulent flows of fluids. Soon he became a leading expert on shock and detonation waves. His interest in hydrodynamics made him a consultant of the Ballistic Research Laboratory of the Army Ordnance Department in 1937, and he became actively engaged in Government work on military matters.

War. When the flames of the Second World War started to engulf Europe, von Neumann was one of the scientists and mathematicians recruited by the United States Government to work on military and defence projects. In 1940, he became a member of the Scientific Advisory Committee of the Ballistic Research Laboratory in Maryland — a position which he held until his death. In 1941, he served on the National Defence Research Committee, and was a consultant of the Navy Bureau of Ordance.

In the wake of the unravelling of the tantalizing secrets of the atomic nucleus came the experimental discovery in 1930 of the splitting of the uranium atom by the German chemists Otto Hahn [28] and Fritz Strassman

[29] , followed in 1934 by the seemingly incredible hypothesis of the chemist Ida Noddack [30] that a heavy nucleus, when bombarded with neutrons, may break up into a number of larger pieces. By the end of 1938, Lise Meitner [31] and her nephew Otto Frisch [32] made the momentous discovery that the breaking up of a uranium nucleus is accompanied by the release of a tremendous amount of energy hitherto undreamt of. The realization that this phenomenon might be reproduced under controlled conditions immediately raised the spectre of an atomic bomb capable of unleasing total destruction. More frightening was the prospect of such a weapon falling into the hands of the Nazis bent on a fanatical mission of mass destruction and subjugation. So, in August, 1942, about eight months after the Japanese attack on Pearl Harbour, the United States Government set up a plan to build an atomic bomb before Nazi Germany could do so. This programme was code-named the "Manhattan Project".

Work on this project was concentrated in three cities: Oak Ridge in Tennesee, Hanford in California, and Los Alamos in New Mexico. A group of brilliant minds was drawn into the top-secret work, and von Neumann spent a large part of 1943-45 at the Los Alamos Scientific Laboratory whose war-time director was the physicist Robert Oppenheimer [33] . In collaboration with the physcists Seth Neddermeyer [34] , Edward Teller [35] and James Tuck [36] , he worked on the theory of implosion as a technique to produce a rapid reaction in a small amount of the uranium isotope, $U^{235}$, or of plutonium, thereby releasing a vast amount of energy. The implosion problem required a numerical solution for which a punched card laboratory was set up, and von Neumann taught the Los Alamos theoreticians how to solve the resulting equations numerically with the aid of the high-speed computer, ENIAC (Electronic Numerical Integrator and Computer)(see [47] ).

The ENIAC, the first electronic digital computer, was built during the early forties in Philadelphia for the Ballistic Research Laboratory and became operational in December, 1945. The man in charge of the ENIAC project, Herman Goldstine [37] met von Neumann in the summer of 1944 and got him interested in the computer project. The participation of von Neumann in discussions with the team charged with the building of the ENIAC marked the beginning of his interest in and contributions (prematurely cut off by his untimely death) to the theory of automata. He helped to design the logical structure of the EDVAC (Electronic Discrete Variable Computer), the immediate successor to the ENIAC.

The potentialities of high-speed computers in solving complex problems of mathematics and mathematical physics had already occurred to von Neumann when he found that the non-linear partial differential equations of hydrodynamics could not be solved analytically and that only numerical solutions could be sought for. He himself possessed a brain that calculated with the speed of a computer. On one occasion, a simple numerical problem was set as a first preliminary test for an electronic computer: what is the smallest power of 2 such that its decimal digit fourth from the right is 7? The computer and von Neumann started together, and he finished first.

Goldstine tells us [47] of an incident which reveals the ease with which von Neumann performs complex calculations in his head. Once a mathematician, who was working on a difficult problem, spent the whole night working out five special cases of increasing complexity on a desk calculator and finished at 4.30 in the morning. Later in the morning, von Neumann visited Goldstine, and the man, who was tired and haggard from the long hours of sleeplessness, was brought in to discuss the problem that was bugging him with von Neumann. After some fruitless discussion, von Neumann decided to work out a few special cases. Unaware of the labour that had been put in before, he worked out in his head four of the previously calculated cases within about five minutes. He had only spent about a minute on the fifth and hardest case when Goldstine's astounded colleague disclosed aloud the answer. Von Neumann thought for about another minute and confirmed it. Whereupon the astounded man fled, leaving von Neumann perplexed and perturbed as to how anyone could have beaten him to it. It was only after the secret was let out that he regained his aplomb.

Post-war. After the war, von Neumann continued to work on mathematical problems related to nuclear research and military strategy (*à la* his theory of games). He thought more and deeper on the theory of computing and the logical theory of automata. He developed methods of numerical analysis in his investigations on numerical meteorology. In November 1945, he became the director of the Electronic Computer Project at the Institute for Advanced Study, which produced the prototype of the modern computer. With his love for puns, he called the computer at Los Alamos the Mathematical Analyser, Numerical Integrator and Computer (MANIAC in short). His collaborators on the Institute computer project affectionately named their new machine JOHNIAC (after von Neumann).

The ending of the Second World War did not reduce the tensions, which had developed in the course of the struggle against Nazi facism, between the rival political systems of the post-war world. The first Russian atomic test of August, 1949, about four years after the first American test at the South Sea atoll of Bikini, caused considerable concern in Washington. Then in January, 1950, Klaus Fuchs [38], who was Deputy Scientific Director at Britain's Atomic Energy Research Establishment at Harwell, was arrested for leaking atomic secrets to the Russians. The shock wave produced in Washington was powerful enough to launch a concerted programme to build a "super" bomb — a thermonuclear bomb which would yield on earth far more energy than imaginable by duplicating the process of fission occurring in the sun. The outbreak of the Korean War in June, 1950, added a new impetus to the drive to build the Super, as the thermonuclear bomb was then called.

Two groups worked independently on the Super: one group headed by Teller at Los Alamos and the other group comprising the Polish mathematician Stanislaw Ulam [39] and his assistant Joseph Everett [40]. The former group was joined by von Neumann with his expertise in handling computers. The physicist Rolf Landshoff [41] mentions a meeting in Teller's office in which quick numerical calculations were made: Richard Feynman [42] on the desk calculator, Enrico Fermi [43] with his slide rule, and von Neumann in his head. "The head was usually first, and it is remarkable how close the three answers always checked." [50]

The first thermonuclear device, nicknamed "Mike", was successfully tested on 1 November 1952 over Eniwetok, one of the tiny atolls of the Marshall Islands in the Pacific. Nine months later, in August, 1953, the Russians carried out a hydrogen bomb explosion which seemed to indicate that they already possessed a more advanced type of thermonuclear bomb. One month later, in September, 1953, a secret committee headed by von Neumann met to discuss plans for a "three-stage" bomb, nicknamed the "fission-fusion-fission" or FFF bomb, which would spread destruction over an area so large as to compensate for the uncertainty of aim of an intercontinental missile.

Von Neumann was closely consulted on nuclear policies. From 1952 to 1954, he served as a member of the General Advisory Committee to the Atomic Energy Commission. In March, 1955, he was sworn in as a member of the Atomic Energy Commission. Then in August, 1955, the severe pains that had developed in his left shoulder was diagnosed after surgery as bone cancer. "The ensuing months", his wife Klara writes [51] , "were of alternating hope and despair". He performed his official duties during the day and worked on his scientific papers during the night. In particular, he worked systematically on the manuscript for the Silliman Lectures [51] which he was invited to deliver at Yale University during the spring of 1956. In November,1955, several lesions were detected on his spine, and by January, 1956, he was confined to a wheelchair. He continued his official and scientific work, and despite medical treatment to sustain him long enough to enable him to fulfil his last academic obligation, he was hospitalized in April,1956. With failing strength, he tried to work on the unfinished manuscript of the Silliman Lectures. He died on 8 February 1957.

<u>Mathematics and beyond</u>. A survey of von Neumann's scientific work has been given by Goldstine and Wigner [52] . His early contributions were made in the axiomatics of set theory and in Hilbert's proof theory. He gave the first axiomatic treatment of a Hilbert space and studied operators in that space. Together with Francis Murray [44] , he founded and developed the theory of rings of bounded linear operators. In connection with this work, he also founded "continuous" geometry. He contributed to the theory of Lie groups, worked on almost periodic functions on groups and gave an elegant proof of the ergodic theorem.

His minimax theorem, proved in a paper of 1928, was a cornerstone in the theory of games. Its applications are given in the well-known book, *Theory of Games and Economic Behaviour*, jointly written with Oskar Morgenstern [45] in 1944. His games of strategy have set a new direction for mathematical economics.

His work on Hilbert spaces, the theory of operators and Lie groups found applications in theoretical physics. His *Mathematische Grundlagen der Quantenmechanik (Mathematical Foundations of Quantum Mechanics)*, written in 1932, became a classic in physics. He also contributed to thermodynamics and statistical mechanics.

In his later years, he helped to develop the modern computer and formulated a theory of coding and programming for machines. He popularized and actively participated in the use of the computer for solving complex problems in mathematics and physics.

Von Neumann was first and foremost a mathematician. As a youth he was already interested in the applications of mathematics to physics. As Goldstine writes [47]: "Unlike many applied mathematicians who want merely to manipulate some equations given them by a physicist, von Neumann would go right back to the basic phenomenon to reconsider the idealizations made as well as the mathematical formulation."

His philosophy of mathematics may be summed up in the following excerpt from the essay, "The mathematician" [53].

"Most people, mathematicians and others, will agree that mathematics is not an empirical science, or at least that it is practised in a manner which differs in several decisive respects from the techniques of the empirical sciences. One of its main branches, geometry, actually started as a natural, empirical science. Some of the best inspirations of modern mathematics (I believe, the best ones) clearly originated in the natural sciences. The methods of mathematics pervade and dominate the 'theoretical' divisions of the natural sciences. In modern empirical sciences it has become more and more a major criterion of success whether they have become accessible to the mathematical method or to the near-mathematical methods of physics. Indeed, throughout the natural sciences an unbroken chain of successive pseudomorphoses, all of them pressing toward mathematics, and almost identified with the idea of scientific progress, has become more and more evident. Biology becomes increasingly pervaded by chemistry and physics, chemistry by experimental and theoretical physics, and physics by very mathematical forms of theoretical physics.

"There is a quite peculiar duplicity in the nature of mathematics. One has to realize this duplicity, to accept it, and to assimilate it into one's thinking on the subject. This double face is the face of mathematics, and I do not believe that any simplified, unitarian view of the thing is possible without sacrificing the essence."

Notes and references

[1] Isaac Newton (1642-1727), English mathematician, physicist and astronomer; invented the differential and integral calculus, founded theoretical mechanics, discovered the law of gravitation. See [46] .

[2] Leonhard Euler (1707-1783), Swiss-born mathematician who spent a large part of his life in St. Petersburg, Russia; a great algorist and analyst. See [46] .

[3] Joseph-Louis Lagrange (1736-1813), French mathematician; well-known for his work on number theory and theoretical mechanics. See [46] .

[4] Carl Friedrich Gauss (1777-1855), German mathematician, astronomer and physicist; exerted a universal influence on mathematics and physics through his contributions to number theory, algebra, geometry, electromagnetism, geodesy. See [46] or C.T. Chong, "Notes on mathematicians, 1. Carl Friedrich Gauss", This *Medley*, Vol.3, No.1 (1975), 6-10.

[5] Henri Poincaré (1854-1912), French mathematician, astronomer and physicist; contributed to number theory, geometry, theoretical mechanics; well-known for his popularization of science. See [46] .

[6] David Hilbert (1861-1943), German mathematician and theoretical physicist; contributed to number theory, set theory, logic, geometry, integral equations and mathematical physics. See [48].

[7] Hermann Weyl (1885 - 1955), Swiss-born mathematician and theoretical physicist; worked in Zürich, Göttingen, Princeton; contributed to number theory, analysis, geometry, algebra, logic, relativity, quantum mechanics.

[8] Norbert Wiener (1894 - 1964), American mathematician and scientist; worked in Cambridge, Göttingen, Massachusetts Institute of Technology; contributed to algebra, analysis, statistical mechanics, physics, cybernetics.

[9] Göttingen is a small German town which became famous for its scientific tradition due to the mathematicians Gauss, Felix Klein (1849 - 1925) and Hilbert. During the first quarter of the present century, it was not only the mecca of German mathematics but also the centre of the scientific world.

[10] Emile Borel (1871 - 1956), French mathematician and politician; worked in the Ecole normale supérieure and the Henri-Poincaré Institute; participated in politics and became a minister in 1925; contributed to analysis and probability.

[11] Bela Kun (1885 - 1937), Hungarian communist leader; journalist and politician; organised an uprising in Budapest on 22 March 1919 and became premier, but ousted on 31 July 1919; spent some years in Russia where he was executed during the purges of the thirties.

[12] German term for a school of the highest order that prepares its students for the university.

[13] Michael Fekete (1886 - 1957), Hungarian-born Israeli mathematician; worked in Budapest, Jerusalem; contributed to theory of functions, Fourier series.

[14] Theodore von Kármán (born 1881), Hungarian-born American scientist; contributed to hydrodynamics, aerodynamics, thermodynamics, engineering.

[15] Fritz Haber (1868 - 1934), German chemist; Nobel prize-winner (1918); contributed to electrochemistry, thermodynamics, synthesis of ammonia.

[16] Erhard Schmidt (1876 - 1959), German mathematician; worked in Berlin; contributed to analysis, topology, number theory.

[17] George Pólya (born 1887), Hungarian-born American mathematician; worked in Göttingen, Zürich, Stanford; contributed to analysis, probability, number theory, applied mathematics.

[18] Latin phrase meaning "with the highest honour or praise".

[19] In the pre-war German academic system, a doctor of philosophy is required to produce another original piece of work for the so-called "Habilitation". He would then be awarded the title of Privatdozent and the privilege to lecture in the university without pay. He could, however, collect fees from students attending his lectures. Upon recognition of his work and abilities, he would receive the salaried post of an Extraordinarius (or assistant professor). The final goal would be an Ordinariat (or full professorship).

[20] Lothar Nordheim (born 1899), German-born American physicist; worked in Göttingen as Hilbert's assistant; now at John Hopkins Laboratory for Pure and Applied Science; contributed to the quantum theory of matter and nuclear physics.

[21] Eugene Wigner (born 1902), Hungarian-born American theoretical physicist; Nobel prize-winner (1963); worked in Berlin, Göttingen, Princeton; contributed to quantum mechanics, nuclear physics.

[22] Paul Ehrenfest (1880 - 1933), Austrian physicist, worked in St. Petersburg (Russia), Leyden; contributed to statistical and quantum mechanics.

[23] Oswald Veblen (1880 - 1960), American mathematician; worked in Chicago, Princeton; contributed to geometry, topology.

[24] James Alexander (born 1888), American mathematician; worked at Cambridge, Princeton; contributed to topology.

[25] Albert Einstein (1879 - 1955), German-born American physicist; Nobel prize-winner (1922); worked in Berne, Zürich, Prague, Berlin, Princeton; contributed to Brownian motion, photo-electricity, special relativity, general relativity.

[26] Marston Morse (born 1892), American mathematician; worked in Harvard, Princeton; contributed to analysis, geometry, topology.

[27] Paul Halmos (born 1916), Hungarian-born American mathematician; worked in Illinois, Princeton, Syracuse, Chicago; now in Bloomington (Indiana); contributed to measure theory, Hilbert space, algebraic logic.

[28] Otto Hahn (born 1879), German chemist; Nobel prize-winner (1944); worked in London, Montreal, Berlin, Göttingen; contributed to radiochemistry.

[29] Fritz Strassman (born 1902), German chemist; contributed to radiochemistry.

[30] Ida Noddack (born 1896), neé Ida Tacke, wife of Walter Noddack (born 1893); German chemist; the Noddacks discovered rhenium in 1925; contributed to chemistry of rare earths.

[31] Lise Meitner (born 1878), German-born Swedish physicist; worked in Berlin, Copenhagen, Stockholm; contributed to radiochemistry.

[32] Otto Frisch (born 1904), German-born British physicist; worked in Harwell, Cambridge; contributed to nuclear physics.

[33] Robert Oppenheimer (1904 - 1967), American physicist; worked in Göttingen, Zurich, Berkeley, Princeton; contributed to nuclear physics.

[34] Seth Neddermeyer (born 1907), American physicist; worked in California, Seattle; contributed to nuclear physics.

[35] Edward Teller (born 1908), Hungarian-born American physicist; worked in Göttingen, Copenhagen, London, Los Alamos, Chicago; now in Berkeley; contributed to quantum theory and nuclear physics.

[36] James Tuck (born 1910), English-born American physicist; worked in Manchester, Oxford; now in Los Alamos; contributed to magnetohydrodynamics, nuclear physics.

[37] Herman Goldstine (born 1913), American mathematician; worked in Chicago, Michigan, Princeton; now with International Business Machines Corporation; contributed to applied mathematics, computer science.

[38] Klaus Fuchs (born 1911), German physicist; worked in Birmingham, Los Alamos; now in Dresden; contributed to nuclear physics.

[39] Stanislaw Ulam (born 1909), Polish-born American mathematician and physicist; worked in Princeton, Harvard, Wisconsin, Los Alamos; contributed to set theory, analysis, logic, topology, mathematical physics.

[40] Joseph Everett (born 1914), American mathematician; worked in Yale, Michigan, Wisconsin, Los Alamos; contributed to algebra, probability.

[41] Rolf Landshoff (born 1911), German-born American mathematical physicist; worked in Los Alamos; now at Lockheed Palo Alto Research Laboratory; contributed to nuclear physics.

[42] Richard Feynman (born 1918), American theoretical physicist; Nobel prize-winner (1965); worked in Princeton, Los Alamos, Cornell; now at California Institute of Technology; contributed to quantum theory, nuclear physics.

[43] Enrico Fermi (1901 - 1954), Italian-born American physicist; Nobel prize-winner (1938); worked in Florence, Rome, Columbia, Chicago; contributed to quantum mechanics, nuclear physics.

[44] Francis Murray (born 1911), American mathematician; worked in Columbia, Durham; contributed to analysis, algebra, applied mathematics.

[45] Oskar Morgenstern (born 1902), German-born American economist; worked in Vienna, Princeton; now at New York University; contributed to mathematical economics.

[46] E.T. Bell, *Men of mathematics*, Simon and Schuster, New York, 1937.

[47] Herman H. Goldstine, *The computer from Pascal to von Neumann*, Princeton University Press, 1972.

[48] Constance Reid, *Hilbert*, George Allen and Unwin, London; Springer-Verlag, Berlin, Heidelberg, New York, 1970.

[49] P.R. Halmos, "The legend of John von Neumann", *American Mathematical Monthly* 80 (1973) 382-394.

[50] Robert Jungk, *Brighter than a thousand suns*, translated by James Cleugh; Gollancz and Hart-Davis, London, 1958; Pelican Books, 1964.

[51] John von Neumann, *The computer and the brain*, Yale University Press, New Haven, 1958.

[52] Herman H. Goldstine and Eugene P. Wigner, "Scientific work of John von Neumann", *Science* 125 (1957), 683-684.

[53] John von Neumann, "The mathematician", in *The works of the mind*, edited by R.B. Heywood, University of Chicago Press, pp. 180-196; reprinted in *Collected works*, Vol.1, No.1, pp.1-9, edited by A.H. Taub; Pergamon Press, 1961, and in *The world of mathematics*, Vol.4, pp.2053-2063, presented by James R. Newman, George Allen and Unwin, 1960.