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Mr. Chong Kim Sing
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Mr. Tan Chek Sian
Mrs. May Tsao

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"The special value of mathematics to biology lies not in its use as a (computational) tool but in its power to abstract and thus lay bare fundamental problems and the relations between superficially distinct entities and processes" -- Edward F. Moore

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# IN-OUT TABLE OF SET THEORY AND TRUTH TABLE OF SYMBOLIC LOGIC

T. H. Kho
Raffles Institution

In symbolic logic, let p, q be two propositions which may be true or false. The Truth Table for the propositions p, q, p *or* q ($p \vee q$), p *and* q ($p \wedge q$). *not* p($\sim p$) is constructed as follows:

Table 1

| p | q | $p \vee q$ | $p \wedge q$ | $\sim p$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 1 |

1 denotes 'the proposition is true' and 0 denotes 'the proposition is false'.

In set theory, let A, B be two sets and a certain object is either in a set or not in a set. An In-Out Table for the sets A, B, A *union* B (A∪B), A *intersection* B (A∩B), *complement of* A (A') is shown below:

Table 2

| A | B | A∪B | A∩B | A' |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 1 |

1 denotes 'a certain object is in the set' and 0 denotes 'a certain object is not in the set'.

One method of proving identities in set theory without using *Venn diagrams* is by the construction of an In-Out Table. For example, given 3 sets A, B and C, to prove

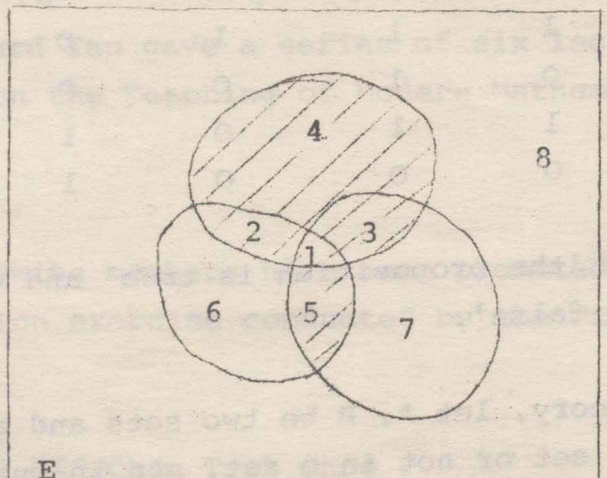
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$$

we construct the following In-Out Table:

Table 3

| | A | B | C | B∩C | A∪(B∩C) | A∪B | A∪C | (A∪B)∩(A∪C) |
|---|---|---|---|---|---|---|---|---|
| (1) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| (2) | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| (3) | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| (4) | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| (5) | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| (6) | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| (7) | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| (8) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

We see that the 5th and 8th columns of the above table are identical and hence the result follows.
Usually this method is interpreted by the use of a Venn diagram.



The universal set F is divided into eight regions, the numbers in the diagram corresponding to the rows of the In-Out Table. We have justified that both the sets $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$ consist of regions 1, 2, 3, 4 and 5, and thus they are equal.

Similarly, the following In-Out Table proves that

$$(A \cup B)' = A' \cap B'$$

Table 4

| A | B | A∪B | (A∪B)' | A' | B' | A'∩B' |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |

We may also give a formal proof for the identities given above. By definitions,

$$x \in A \cup B \iff x \in A \text{ or } x \in B \ ,$$

$$x \in A \cap B \iff x \in A \text{ and } x \in B \ ,$$

and

$$x \in A' \iff \text{not } x \in A \iff x \notin A \ .$$

In the first example, we need to prove

$$x \in A \cup (B \cap C) \iff x \in (A \cup B) \cap (A \cup C).$$

We may proceed as follows:

$$x \in A \cup (B \cap C)$$

$$\iff x \in A \text{ or } x \in B \cap C$$

$$\iff x \in A \text{ or } (x \in B \text{ and } x \in C)$$

$$\overset{\circledast}{\iff} (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$$

$$\iff x \in A \cup B \text{ and } x \in A \cup C$$

$$\iff x \in (A \cup B) \cap (A \cup C) \ .$$

The step ⊛ in the proof can be justified if we let the propositions

$p \equiv$ '$x \in A$', $q \equiv$ '$x \in B$', $r \equiv$ '$x \in C$'

and verify that

$$p \vee (q \wedge r) \qquad (p \vee q) \wedge (p \vee r) \ .$$

To verify this we construct the following Truth Table:

Table 5

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | $\underline{0}$ | 0 | 0 | $\underline{0}$ |

The 5th and 8th columns of the above table show that the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are equivalent and hence the result follows.

It is not difficult to observe that Table 3 is an alternative form of Table 5. This provides a logic-theoretic interpretation of the In-Out Table of set theory.

The second example can be resolved as follows:

$$x \in (A \cup B)'$$

$$\iff x \notin A \cup B$$

$$\iff \textit{not} \quad x \in A \cup B$$

$$\iff \textit{not} \quad (x \in A \quad \textit{or} \quad x \in B)$$

$$\overset{⊛⊛}{\iff} (\textit{not}\ x \in A) \ \textit{and}\ (\textit{not}\ x \in B)$$

$$\iff x \notin A \quad \textit{and} \quad x \notin B$$

$$\iff x \in A' \ \textit{and} \ x \in B'$$

$$\iff x \in A' \cap B',$$

i.e. $\quad x \in (A \cup B)' \iff x \in A' \cap B'$.

The step ⊛⊛ is justified by verifying that

$$\sim(p \vee q) = (\sim p) \wedge (\sim q)$$

from the following Truth Table.

Table 6

| $p$ | $q$ | $p \vee q$ | $\sim(p \vee q)$ | $\sim p$ | $\sim q$ | $(\sim p) \wedge (\sim q)$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | $\underline{1}$ | 1 | 1 | $\underline{1}$ |

Again, it can be observed that Table 4 is an alternative form of Table 6.

The readers may try to prove the *duals* of the above two examples, namely,

$$A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$$

and

$$(A\cap B)' = A'\cup B',$$

which are obtained by interchanging $\cup$ and $\cap$.

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When G.M. Hardy visited S. Ramanujan he told him that the number of the taxi in which he came, 1729, looked rather unattractive. Ramanujan immediately denied this, saying that it was the least number which could be expressed as the sum of two cubes in two different ways ; that is, $1729 = 12^3 + 1^3 = 10^3 + 9^3$.

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# EQUAL LIKELIHOOD AND INDEPENDENCE

Louis H. Y. Chen
Department of Mathematics
University of Singapore

1. *Introduction.* The notion of equal likelihood is, in some sense, closely tied with that of independence in the Theory of Probability. Often this fact is either overlooked or insufficiently emphasized in the teaching of elementary courses. This is perhaps due to the difficulty in making the relation between the two notions precise at the level concerned. However, a good understanding of the relation is necessary in order to explain the equivalence of two methods which are often employed in the solutions of a large number of elementary problems.

To illustrate the last point, consider the following simple example: A fair coin is tossed, a fair die is rolled and a ball is drawn at random from an urn containing 2 black and 8 white balls. What is the probability of the event that the coin falls heads, an even number appears on the die and a white ball is drawn? One method of solution is as follows: Since the sample space consists of 2×6×10 = 120 equally likely outcomes (each of which can be represented by a triple such as (Head,5,black ball)) and the event consists of 1×3×8 = 24 outcomes, it follows that the probability is 24/120 = 1/5. Alternatively, one could first calculate the probabilities of the coin falling heads, of an